1. The ball of mass $m$ is suspended by cables $A$ and $B$. Cable $B$ is cut. Is the force in cable $A$ going to increase or decrease? Explain (5 points)

\[ \sum F_x = 0 \Rightarrow T_A = 0 \]

\[ \therefore T_A = mg \cos 45^\circ \]

2. Derive the principle of work and energy for a single particle. (5 points)

\[ a_t \, ds = v \, dv \]

\[ \sum F_x \, ds = m \, v \, dv \]

\[ \int_{A}^{B} F \, ds = m \int_{A}^{V_B} v \, dv = \frac{1}{2} m v_B^2 \]

\[ \frac{m v_A^2}{2} + \int_{A}^{B} F \, ds = \frac{m v_B^2}{2} \]

\[ \frac{1}{2} T_A + U_{AB} = \frac{1}{2} T_B \]
3. Use the definition of work to compute the work of a constant force $F = (-1, 3)$ [kN] along the circular segment $AB$ (5 points)

$$\text{Parametrization: } \begin{cases} x = 2 \sin \theta \\ y = -2 \cos \theta \end{cases}$$

$$\int_{A}^{B} F \cdot dl = \int_{0}^{\frac{\pi}{2}} \left( F_{x} \frac{dx}{d\theta} + F_{y} \frac{dy}{d\theta} \right) d\theta$$

$$\int_{A}^{B} = \int_{0}^{\frac{\pi}{2}} (-2 \cos \theta + 6 \sin \theta) d\theta = \left[-2 \sin \theta - 6 \cos \theta\right]_{0}^{\frac{\pi}{2}} = 4$$

4. Derive the principle of angular impulse and momentum for a single particle. (5 points)

$$0 \text{- fixed}$$

$$\mathbf{r} \times \mathbf{F} = \mathbf{r} \times \mathbf{F}$$

$$(\mathbf{r} \times \mathbf{m} \mathbf{\dot{v}}) = \mathbf{r} \times \mathbf{F}$$

$$\mathbf{r} \times \mathbf{m} \mathbf{\dot{v}}$$

Angular momentum with 0

5. Identify the IC of zero velocity for the bar BC. (5 points)
6. Suppose you are designing a roller-coaster track that will take cars through a vertical loop of 40-ft radius. If you decide that, for safety, the downward force exerted on a passenger by his or her seat at the top of the loop should be at least one-half the passenger’s weight, what is the minimum safe velocity of the cars at the top of the loop? (25 points)

\[ N - \text{force exerted by the seat on the passenger} \]

Notice that we must have \( N \geq 0 \), if we do not want to rely exclusively on seat belts...

For safety, \( N \geq \frac{1}{2} mg \)

Eqn. of motion in the normal direction

\[ m \frac{v^2}{r} = N + mg \]

So

\[ m \frac{v^2}{r} - mg = N \geq \frac{mg}{2} \]

\[ \frac{v^2}{r} \geq \frac{\frac{3}{2} mg}{r} \]

\[ v^2 \geq \frac{3}{2} gr \]

\[ v \geq \sqrt{\frac{3}{2} gr} = 43.95 \text{ ft/s} \]
7. A ball is given a horizontal velocity of 4 m/s at 2 m above the smooth floor. Determine distance \( D \) between the ball's first and the second bounces if the coefficient of restitution is \( e = 0.6 \). (25 points)

**Step 1:** Motion before the first impact

\[
\begin{align*}
\dot{x} &= 4 \\
\dot{y} &= -9.81t \\
\end{align*}
\]

\( y = 0 \Rightarrow t = 0.639 \text{ [s]} \)

**Step 2:** Impact

Velocity of the ball before impact

\[
\begin{align*}
v_x & = 4 \text{ m/s} \\
v_y & = -6.27 \text{ m/s} \\
\end{align*}
\]

Velocity of the ball after impact

\[
\begin{align*}
v_x & = v_x = 4 \text{ m/s} \\
v_y & = -v_y \Rightarrow v_y = +3.76 \text{ m/s} \\
\end{align*}
\]

**Step 3:** Motion between the impacts

Shifting coordinates to the plane of first impact (A)

\[
\begin{align*}
x &= 4t \\
y &= 3.76t - \frac{9.81t^2}{2} = 0 \Rightarrow t = 0.767 \text{ [s]} \\
\end{align*}
\]

(new \( t \), measured from the time of the first impact)

\[
D = x = 3.07 \text{ m}
\]
8. Bar $AB$ has an angular velocity of 4 rad/s in the clockwise direction and an angular acceleration of 10 rad/s$^2$ in the counterclockwise direction. What is the acceleration of pin $B$ relative to the slot? (25 points)

Velocities:

$$\gamma_B = \dot{x}^0_x + \dot{x}_{AB} \times AB = \frac{x_{AB} (0, 0, -4)}{AB (0.115, 0.06, 0)}$$

$$= (0.24, -0.46, 0) \left[ \frac{m}{s} \right]$$

$$\gamma_B = \dot{x}^0_c + \dot{x}_{CB} \times CB + \gamma_{rel} = x_{CB} (0, 0, \omega_{CB}) \frac{CB (0.035, 0.06, 0)}{(-0.06 \omega_{CB}, 0.035 \omega_{CB}, 0)} + (v_{rel}^{\text{rel}}, 10)$$

Comparing:

$$0.24 = -0.06 \omega_{CB} + v_{rel}^{\text{rel}}$$

$$-0.46 = 0.035 \omega_{CB} \Rightarrow \omega_{CB} = -13.14 \frac{\text{rad}}{s}$$

$$\Rightarrow v_{rel}^{\text{rel}} = -0.549 \frac{m}{s}$$

Accelerations:

$$a_B = \ddot{x}^0_x + \ddot{x}_{AB} \times AB - \dot{x}_{AB}^2 \times AB = \frac{x_{AB} (0, 0, 10)}{AB (0.115, 0.06, 0)} \left( \frac{-16 (0.115, 0.06, 0)}{(-0.6, 1.15, 0)} \right)$$

$$= (-2.44, 0.19, 0) \left[ \frac{m}{s^2} \right]$$

$$a_B = \ddot{x}^0_c + \ddot{x}_{CB} \times CB - \dot{x}_{CB}^2 \times CB + a_{rel}^{\text{rel}} + 2 \dot{x}_{CB} \times \dot{v}_{rel}$$

$$= \frac{x_{CB} (0, 0, \omega_{CB})}{CB (0.035, 0.06, 0)} \left( -0.06 \omega_{CB}, 0.035 \omega_{CB}, 0 \right) - (13.14)^2 (0.035, 0.06, 0)$$

$$+ (a_{rel}^{\text{rel}}, 10) + 2 \frac{x_{CB} (0, 0, a_{rel}^{\text{rel}} - 13.14)}{v_{rel}^{\text{rel}} (1.028, 0, 0)}$$

$$= (0, -13.5, 0)$$
Comparing accelerations:

\[-2.44 = -0.06 \alpha_{cB} - 6.04 + \alpha_{Bx}^{1st}\]

\[0.19 = 0.035 \alpha_{cB} - 10.35 + 14.42\]

\[\alpha_{cB} = -110.9 \frac{\text{m}}{\text{s}^2}\]

\[\alpha_{Bx}^{1st} = -3.05 \frac{\text{m}}{\text{s}^2}\]