Homework set 14 — APPM5450

From the textbook: 13.2, 13.3, 13.10.

Problem: Set $I = [0, 1]$, $X = L^2(I)$, $Y = L^1(I)$, and consider the map

$$f : X \to Y : u \mapsto u^2.$$

(a) Prove that f is continuously differentiable, and calculate f'.

(b) Set $\hat{u} = 1 \in X$, and $\hat{v} = f(\hat{u})$. Prove that there cannot exist open sets $G \subset X$, and $H \subset Y$, such that $\hat{u} \in G$, $\hat{v} \in H$, and a map $g : H \to G$ such that

$$f(g(v)) = v, \forall\ v \in H,$$

and

$$g(f(u)) = u, \forall\ u \in G.$$

(c) Why do (a) and (b) together not contradict the inverse function theorem?