Homework set 11 — APPM5440 — Fall 2016

From the textbook: 5.11, 5.12, 5.13, 5.15a, (5.16), 5.17.

Problem 1: Let X be a Banach space, and let $A, B \in \mathcal{B}(X)$ be two operators such that $AB = BA$. Prove that $e^{A+B} = e^A \, e^B$.

Problem 2: For $n = 1, 2, 3, \ldots$, we define the operator T_n on $X = l^2(\mathbb{N})$ by
$$T_n(x_1, x_2, x_3, \ldots) = \frac{1}{\sqrt{n}}(x_1, x_2, \ldots, x_n, 0, 0, \ldots).$$
Prove that $T_n \in \mathcal{B}(X)$. Does T_n converge to anything in norm? Strongly?