Notes on Midterm 3 – APPM5440 – Fall 2005:

The exam covers chapters 5 and 6. For chapter 6, the syllabus is defined by the lecture notes distributed (the material on projections in Banach spaces is *not* included). In Chapter 5, the following topics are important:

- Definition of a Banach space. The spaces l^p, $C(I)$, $C^k(I)$.
- The space $\mathcal{B}(X, Y)$. The operator norm (equations (5.2) and (5.3) are important). Strong convergence. Norm convergence implies strong convergence.
- For a linear operator: Continuity $\Leftrightarrow$ Boundedness. (Thm. 5.18)
- Isomorphisms between Banach spaces.
- Equivalent norms.
- Statement of the open mapping theorem.
- Properties of the kernel and the range of a linear operator. Coercive operators have closed range (Prop. 5.30).
- Simplifications in finite-dimensional spaces (all linear operators are bounded, all norm topologies are equivalent, *etc*).
- Theorem 5.37 ($||ST|| \leq ||S||\,||T||$).
- Definition of a compact operator. Prop. 5.43.
- Definition of the (topological) dual X^* of a normed linear space X. Norm convergence and weak convergence in X^*.
- The Hahn-Banach theorem. The linear functionals separate points in X. The elements of X separate points in X^* (so that the weak-$*$ topology on X^* is Hausdorff).

The following material is included in the syllabus but is not core material (you are not expected to have it memorized):

- Extension of a bdd linear operator defined on a dense set (Thm 5.19).
- Definition of the exponential of an operator.
- Weak-$*$ convergence in X^*. Alaoglu's theorem. Isometric embedding of X into X^{**}.