L-curve-based regularization parameter selection
Antarctic ice sheet inversion for basal sliding field: InSAR data

Left: InSAR-based Antarctica ice surface velocity observations
Right: Inferred basal sliding field

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Reconstructed ice surface velocity field
Invert for log-coefficient field $\gamma(x)$ in Poisson equation from observations of state $u(x)$ within domain:

$$\min_{\gamma \in \mathcal{M}} \frac{1}{2} \int_0^L \left( b u(\gamma) - d_{\text{obs}} \right)^2 dx$$

where the map from $\gamma$ to $u$ is defined by solution of:

$$- \frac{d}{dx} \left( e^{\gamma} \frac{du}{dx} \right) = 0 \quad \text{for } x \in (0, L)$$

$$u(0) = u_0$$

$$u(L) = u_L$$

Two cases of observation operator:

- full observations: $b(x) = 1$
- pointwise observations: $b(x) = \frac{L}{n} \sum_{j=1}^n \delta(x - x_j)$
assume no noise, $\gamma = \gamma_0$ is true log-medium

Fourier analysis of spectrum of data misfit Hessian, evaluated at $\gamma_0$, gives eigenvalues for both cases:

- full observations: $\lambda_i = \frac{(u_L - u_0)^2}{\pi^2 i^2}$
- $n$ pointwise observations: $\lambda_i = \frac{(u_L - u_0)^2}{4n^2 \sin(i\pi/2n)^2}$

Spectrum of data misfit Hessian for both full and pointwise observations

Left: 6 observations  
Right: 500 observations
Eigenfunctions of data misfit Hessian are $\cos(i\pi x/L)$ in full observations case, and piecewise-constant interpolants of the cosines in pointwise case.

Eigenfunctions 1, 2, 5, and 6 for full and pointwise ($n = 6$) observations.
Spectrum of the prior-preconditioned data misfit Hessian

- Spectrum of $\Gamma_{pr}^{1/2} F^T \Gamma_{\text{noise}}^{-1} F \Gamma_{pr}^{1/2}$ for Antarctica inverse problem with 410K and 1.19M basal sliding parameters (observed to decay like $i^{-3}$)
- 4000 dominant modes, independent of parameter and data dimension
- Intrinsic problem dimension depends on information content of data
Eigenvector 1 of prior-preconditioned data misfit Hessian
Eigenvector 2 of prior-preconditioned data misfit Hessian
Eigenvector 3 of prior-preconditioned data misfit Hessian
Eigenvector 4 of prior-preconditioned data misfit Hessian
Eigenvector 5 of prior-preconditioned data misfit Hessian
Eigenvector 6 of prior-preconditioned data misfit Hessian
Eigenvector 7 of prior-preconditioned data misfit Hessian
Eigenvector 8 of prior-preconditioned data misfit Hessian
Eigenvector 9 of prior-preconditioned data misfit Hessian
Eigenvector 10 of prior-preconditioned data misfit Hessian
Eigenvector 100 of prior-preconditioned data misfit Hessian
Eigenvector 200 of prior-preconditioned data misfit Hessian
Eigenvector 500 of prior-preconditioned data misfit Hessian
E-vector 1000 of prior-preconditioned data misfit Hessian
E-vector 2000 of prior-preconditioned data misfit Hessian
E-vector 4000 of prior-preconditioned data misfit Hessian