THERMO-VISCOPLASTIC ANALYSIS OF HYPersonic
STRUCTURES SUBJECTED TO SEVERE
AERODYNAMIC HEATING

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The commitment to develop the National Aerospace Plane has generated resurgent interest in the technology required to design structures for hypersonic flight. Such structures will be exposed to aerodynamic heating of unprecedented magnitudes. As the vehicle accelerates or decelerates at hypersonic speeds in the atmosphere, shocks will sweep across the vehicle and interact with local shocks and boundary layers. These interactions introduce severe local pressures and heating rates. A recent experimental study (ref. 1) of interacting shock waves on a cylindrical leading edge shows heating rates ten times undisturbed levels.

Leading edges of engine structures present a significant design problem because of intense local heating and pressures. Analysis of the flow, thermal and structural behavior present serious computational challenges to analysts because of the inherent nonlinearities in all aspects of the multi-disciplinary problems. Some of the critical computational issues are identified in reference 2. Critical issues include the difficulties involved in (1) analyzing the viscous, compressible flow and predicting the high local aerodynamic heating, (2) modeling and analyzing multi-mode unsteady heat transfer in a high temperature convectively-cooled structure, and (3) simulating the transient, nonlinear thermal-structural response for rapid temperature changes. Preliminary structural analysis of an impingement cooled leading edge (ref. 3) showed high local plasticity that seriously degraded the structure's load carrying capacity at elevated temperatures. A recent thermostructural analysis with experimental verification (ref. 4) of cowl lip designs confirmed that inelastic effects occur and can be significant. In the experimental study, two specimens failed due to burn-through because of intense local heating or because of loss of cooling.

The purpose of this paper is to present a thermo-viscoplastic computational method for hypersonic structures subjected to severe local unsteady heating. The analysis employs a unified visco-plastic constitutive model implemented in a finite element approach capable of predicting rate-dependent plasticity effects for temperatures up to about 75% of the melting point. Rate-dependent plasticity effects are known to be important at elevated temperatures. The balance of this abstract highlights the unified viscoplasticity theory, outlines the computational approach and presents preliminary results for two thermo-viscoplastic analyses of a convectively cooled structure.

Unified visco-plastic constitutive models have evolved over the last twenty years to provide a means for analytically representing a materials response from the elastic through the
plastic range including strain-rate dependent plastic flow, creep and stress relaxation. The theories are guided by physical considerations including dislocation dynamics and are based on the principles of continuum mechanics. The first multi-dimensional formulations of elastic-viscoplastic constitutive equations was due to Bodner and Partom. Since then a number of constitutive models have appeared; many of these theories are summarized in review articles that appear in reference 5. A NASA-Lewis sponsored research program (HOST) conducted by the Southwest Research Institute recently concluded a four year research effort (ref. 6-7) to further develop unified constitutive models for isotropic materials and to demonstrate their usefulness for analysis of high temperature gas turbine engines. One result of this study is material property data for high temperature nickel-based alloys over a wide temperature range. The unified models employed were those of Bodner-Partom and Walker.

Unified visco-plastic theories have been implemented by a number of finite element researchers. Under the NASA HOST program, the Walker model was implemented in the MARC finite element program (ref. 8) and used to analyze the thermo-viscoplastic response of a turbine blade under simulated flight conditions. In another recent finite element application (ref. 9), the Bodner-Partom and Walker theories were compared for a thin circular plate subject to highly localized, transient heating.

In this paper the Bodner-Partom constitutive model is employed, and the finite element approach developed in reference 10 for the isothermal case is extended to include thermal effects.

The behavior of a thermo-viscoplastic structure subjected to aerodynamic heating is analyzed assuming that: (1) thermo-mechanical coupling in the conservation of energy equation can be neglected, (2) the structural response is quasi-static, and (3) deformations are infinitesimal. With these assumptions, an unsteady thermal analysis may be performed first to determine the temperatures. Then, using these temperatures, the structure's viscoplastic response is determined. The solution is thus obtained by separately solving an initial-boundary value problem for first the thermal and then the structural response.

For hypersonic flight, some leading edges and panels require active cooling systems to keep structural temperatures within acceptable ranges. The internal flow in the coolant passage has a predominant role in the thermal response of a hypersonic structure subject to external heating. The paper will describe an engineering heat transfer approach (ref. 11) that is used to model heat transfer in a convectively cooled structure.

The viscoplastic initial value problem is formulated in terms of stresses, strains and displacements expressed in rate form. The governing equations, boundary and initial conditions are summarized in Figure 1. The figure shows that strain rates are separated into elastic and
plastic components. The stress rates are related to the elastic strain components and temperature rates by Hooke's law. The plastic strain rates are related to stresses and an internal state variable by the unified viscoplastic theory.

The Bodner-Partom constitutive equations are summarized in Figure 2. Note that the state variable \( z \) is given by an evolution equation which must be integrated in time as part of the analysis.

The finite element formulation for the thermal and structural problem will be presented in the paper. The finite element solution method is summarized in Figure 3, and steps in the thermo-viscoplastic solution scheme are summarized in Figure 4.

Some preliminary numerical results for a segment of a convectively cooled scramjet engine strut are presented in Figures 6-11. The figures presented show the thermal and viscoplastic response for a one-D model (Figs. 6-7) and a two D model (Figs. 8-11). Results include transient temperatures, stresses, deformations and plastic strains. The paper will discuss the problem in detail and present additional results.

The preliminary results show that localized, transient aerodynamic heating caused significant plasticity in the aerodynamic skin. The structural response was strongly influenced by the temperatures and the temperature rates of change. Residual stresses and permanent deformations can be predicted without excessive computational expense because the coolant flow causes a short duration transient. The paper will demonstrate that thermo-viscoplastic analysis can provide valuable insight into the behavior of hypersonic structures subject to severe aerodynamic heating.

REFERENCES


8. MARC General Purpose Finite Element Program, MARC Corporation, Palo Alto, CA.


INITIAL VALUE VISCOELASTICITY PROBLEM

· EQUILIBRIUM (RATE FORM): \[ \dot{\sigma}_{ij,j} + \dot{\beta}_j = 0 \]

· KINEMATICS: \[ \dot{E}_{ij} = \dot{E}_{ij}^E + \dot{E}_{ij}^P = \frac{1}{2} (\dot{u}_{i,j} + \dot{u}_{j,i}) \]

· CONSTITUTIVE: \[ \dot{\sigma}_{ij} = E_{ijkl} \dot{E}_{kl}^E - E_{ijkl} \delta_{kl} \Delta T \]
  \[ \dot{E}_{ij}^P = f(\sigma_{ij}, T) \]
  \[ \dot{T} = g(\sigma_{ij}, T) \]

· BOUNDARY CONDITIONS: \[ \dot{u}_i = \ddot{u}_i \quad \text{on } \partial \Omega_1 \]
  \[ \dot{\sigma}_i = \sigma_{ij} n_j \quad \text{on } \partial \Omega_2 \]

· INITIAL CONDITIONS: \[ \text{SPECIFY, } \dot{u}_i (X, 0) \]
  \[ \sigma_{ij} (X, 0) \]
  \[ T(0) = T_0 \]

Figure 1 - Thermo-viscoelastic initial-boundary value problem
**Bodner-Partom Constitutive Equations**

**Strain Rates:**
\[ \dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^E + \dot{\epsilon}_{ij}^P \]

**Deviatoric Stresses:**
\[ S_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk} \]

**Deviatoric Stress Invariant:**
\[ J_2 = \frac{1}{2} S_{ij} S_{ij} \]

**Elastic Stress-Strain-Temperature:**
\[ \dot{\sigma}_{ij} = E_{ijkl} \dot{\epsilon}^E_{kl} - E_{ijkl} \alpha_{kl} \Delta T \]

**Plastic Strain Rate:**
\[ \dot{\epsilon}_{ij}^P = \frac{S_{ij}}{\sqrt{J_2}} D_0 \exp \left[ -\frac{1}{2} \left( \frac{Z}{3J_2} \right)^n \right] \]

**Internal State Variable:**
\[ \dot{Z} = m_l (Z_l - Z) \dot{W}_P - A_1 Z_l \left( \frac{Z - Z_l}{Z_l} \right)^{r_l} \]
\[ Z(0) = Z_0 \]
PLASTIC WORK: \[ \dot{W}_p = \sigma_{ij} \dot{e}_{ij} \]

MATERIAL CONSTANTS: \[ m_1, z_1, \gamma, D_0 \]

TEMPERATURE-DEPENDENT
MATERIAL PARAMETERS: \[ n(T), z_0(T), A_1(T), z_2(T) = z_0 \]
\[ E(T), G(T), \alpha(T) \]

DATA AVAILABLE FOR: \( Ti, Cu, Al \)
\( Al \ 2024-0 \)
\( Rene' 95 \)
\( IN-100 \)
\( INCONEL 718 \)
\( Hastelloy-X \)

* B1900 + Hf
* MAR-M 247
* USED IN CALCULATIONS

Figure 3 Bodner-Partom Constitutive Equations (Concluded)
**Finite Element Solution Method**

1. **Solve Transient Thermal Problem**
   - **Time Step** \( \Delta t_T \)
   - Obtain \( \{ T \}_t_1, \{ T \}_t_2, \ldots \)

2. **Solve Initial Statics Structures Problem** if \( T(x,0) \neq T_{ref} \)
   - Obtain initial displacements and stresses

3. **Time-March Viscoelastic Solution**
   - **Time Step** \( \Delta t_s < \Delta t_T \)
   - Interpolate temperatures
   - Solve F.E. equilibrium eq. for \( \{ \dot{\delta} \} \) at each step
   - Time March element quantities to next time
     - \( \{ \sigma \}, \{ \varepsilon \} \)
     - Evaluated at Gauss points

*Figure 4 - Finite Element Solution Method*
THermo-ViSCoPlastic Solution

1. AT TIME $t$, INITIalize $\sigma_{ij}, z$ FOR EACH ELEMENT

2. CALCULATE $\dot{\sigma}_{ij} = f(\sigma_{ij}, z)$ FOR EACH ELEMENT

3. ASSEMBLE AND SOLVE $[K]\{\dot{\epsilon}\} = \{F\}$

4. CALCULATE $\dot{\epsilon}_{ij}$ FOR EACH ELEMENT, $\{\dot{\epsilon}\} = [B]\{\dot{u}\}$

5. CALCULATE $\dot{\sigma}_{ij}$ FOR EACH ELEMENT, $\{\dot{\sigma}\} = [E]\{\dot{\epsilon} - \dot{\epsilon}^p\} - [E] \cdot \dot{u}$

6. CALCULATE $\dot{z}$ FOR EACH ELEMENT, $\dot{z} = g(\sigma_{ij}, z)$

7. INTEGRATE $\dot{\sigma}_{ij}, \dot{z}$ FORWARD FOR EACH ELEMENT TO GET $\sigma_{ij}$ AND $z$ AT $t + \Delta t$

8. IF $t + \Delta t < t_{FINAL}$ GO TO 2

9. STOP

Figure 5 - Steps in Thermo-Viscoplastic Solution
1-D MODEL

cooling (665 R)

heating (5345 R)

radiation

Figure 6  1-D Model of Convectively-Cooled Structure
Figure 7  Thermo-Viscoplastic Response of 1D Model
FINITE ELEMENT MODEL

Figure 8  2D Model of Convectively-Cooled Structure
THERMAL PROBLEM

TEMPERATURE HISTORY ON THE SKIN

TEMPERATURE CONTOURS AT .5 sec

$T_{\text{max}} = 2053 \, \text{R}$

Figure 9 Thermal Response of 2D Model
Figure 10  Aerodynamic skin viscoelastic stress response.
SCRAMJET STRUT

$u_{\text{max}} = 0.0006 \text{ in}$

DEFORMED MESH AT .5 sec

$\varepsilon_{\text{max}} = 0.2\%$

PRINCIPAL PLASTIC STRAIN AT .5 sec

Figure 11 2D Viscoplastic Structural Response at $t=0.5\text{sec}$.