Broadband Sensor Location Selection Using Convex Optimization in Very Large Scale Arrays

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Parabolic audio dish: can we do better?
Adaptive target selection?
Motivation

Adaptive target selection?
Motivation

Industrial applications

Motivation

Industrial applications
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Industrial applications
Outline

1) Very Large Microphone Arrays
2) Problem Setup
3) Optimization Problem
4) Full Routine
5) Experimental Results
1) Very Large Microphone Arrays
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Very Large Microphone Arrays

1) Hardware is **inexpensive** (10,000 microphones feasible)

2) **Data** from all microphones is overwhelming

3) Very large microphone arrays have **great diversity**
Hardware *inexpensive* (10,000 microphones feasible)

e.g. line entire perimeter with mics
Data from all microphones **overwhelming**

10000 sensors x
(44100 samples/sec) x
(16 bits/sample)

\[ \approx 6729 \text{ Mb/sec} \]
Very Large Microphone Arrays

VLMAs have great diversity

10000 possible mics

300 mics used

\[ \approx 10^{583} \text{ possibilities} \]
Outline

1) Very Large Microphone Arrays

2) Problem Setup

3) Optimization Problem

4) Our Algorithm

5) Experimental Results
Problem Setup

- 1000 mics on perimeter of 10 m x 8 m room
  → choose subset of 32 mics

- Few actual sources with unknown location

- Minimize max gain of 6200 locations

- Assume sources consist of 4 frequencies
  (250, 500, 750, 1000 Hz)

[Bertrand&Moonen,2010, Sensor Selection]
minimize $\max \text{ gain}$ over green with target unit gain, assume no sources in red or cyan

[Brandstein&Ward 2000, Beamforming-scanning]
Outline

1) Very Large Microphone Arrays (VLMA)

2) Problem Setup

3) Optimization Problem

4) Full Routine

5) Experimental Results
Optimization Problem

1) Implement frequency dependent delay, scale, and sum beamformer

2) Optimize weights (delays) of all mics to minimize max interference gain

3) Add $L_1$ penalty of weights to optimization criteria to find sparse mic subset
Optimization Problem

\[
\text{min max "broadband" gain across "L" interferences}
\]

\[
\text{with sparse weights}
\]

subject to unit target gain at each frequency
Optimization Problem

$$\min_{(w_n(f_i))} \max_{1 \leq l \leq L} \sum_{i=1}^{F} \left| \sum_{n=1}^{N} H_{ln}(f_i) w_n(f_i) \right|$$
Optimization Problem

\[
\min \left( w_n(f_i) \right) \quad \forall i \in \{1, \ldots, F\}
\]

\[
\max \left( \sum_{i=1}^{L} \left| \sum_{n=1}^{N} H_{ln}(f_i) w_n(f_i) \right| \right)
\]

\[
F = 4 \text{ frequencies (250, 500, 750, 1000 hz)}
\]

\[
L = 6200 \text{ interferences}
\]

\[
N = 1000 \text{ mics}
\]
Example: (fixed weights, fixed $\lambda$)

$f_1 = 250 \text{ hz}$

$f_2 = 500 \text{ hz}$

$f_3 = 750 \text{ hz}$

$f_4 = 1000 \text{ hz}$

add pointwise
Example: (fixed weights, fixed $\lambda$)

$$\max_{1 \leq l \leq L} \sum_{i=1}^{F} | \sum_{n=1}^{N} H_{ln}(f_i) w_n(f_i) |$$

max "broadband" gain
Optimization Problem

\[ \lambda \sum_{n=1}^{N} \max_{1 \leq i \leq F} |w_{n}(f_{i})| \]

with sparse weights
Optimization Problem

\[ \lambda \text{ weighted } L_1 \text{ penalty term that induces sparsity of } w_n \text{ vectors} \]

\[ \lambda \sum_{n=1}^{N} \max_{1 \leq i \leq F} |w_n(f_i)| \]

with sparse weights

\[ \lambda \text{ tuning parameter (larger, more sparse)} \]
subject to \( \sum_{n=1}^{N} H_{0n}(f_i)w_n(f_i) = 1 \) for \( i = 1, 2, \ldots, F \)

subject to unit target gain at each frequency
Optimization Problem

\[
\min_{w_n(f_i)} \max_{1 \leq l \leq L} \sum_{i=1}^{F} \left| \sum_{n=1}^{N} H_{ln}(f_i)w_n(f_i) \right| + \lambda \sum_{n=1}^{N} \max_{1 \leq i \leq F} \left| w_n(f_i) \right|
\]

subject to \( \sum_{n=1}^{N} H_{0n}(f_i)w_n(f_i) = 1 \) for \( i = 1, 2, \ldots, F \)

with sparse weights

subject to unit target gain at each frequency

[Lebret&Boyd 1997, Convex Optimization]
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Run *multiple* optimizations, *varying* $\lambda$, until we find desired number of significant weights (e.g. 32).

\[
\min_{\{w_n(f_i)\}_{i=1}^F} \max_{1 \leq l \leq L} \sum_{i=1}^{F} \left| \sum_{n=1}^{N} H_{ln}(f_i) w_n(f_i) \right| + \lambda \sum_{n=1}^{N} \max_{1 \leq i \leq F} |w_n(f_i)|
\]

subject to $\sum_{n=1}^{N} H_{0n}(f_i) w_n(f_i) = 1$ for $i = 1, 2, \ldots, F$

[Tibsharani 1996, LASSO]
Step 2 of 2 [Weight Discovery]

Run optimization problem over discovered subset in **Step 1 with no sparsity penalty** (debias)

\[
\min_{(w_n(f_i))} \max_{1 \leq l \leq L} \sum_{i=1}^{F} \sum_{n=1}^{N} H_{ln}(f_i)w_n(f_i) \\
\text{subject to } \sum_{n=1}^{N} H_{0n}(f_i)w_n(f_i) = 1 \text{ for } i = 1, 2, \ldots, F
\]
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Comparison Algorithms

1) Optimal Uniform Linear Array (OULA)
2) Multi-Frequency $\lambda$ – proposed
3) Simulated Annealing (SA)
Optimal Uniform Linear Array

Worst Gain: 10.4 dB (1000 Hz)

Microphones closest to target, maximizes SNR
Multi-Frequency \( \lambda \)

Worst Gain: 2.4 dB (1000 Hz)

32 microphones out of 1000 for 4 frequency optimization
Simulated Annealing

Worst Gain: 5.8 dB (1000 Hz)

Random search, accept worse subset with decreasing probability
Worst Gain Comparisons (dB)

<table>
<thead>
<tr>
<th>f[Hz]</th>
<th>OULA</th>
<th>Multi-frequency λ method</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>15.6</td>
<td>9.2</td>
<td>3.9</td>
</tr>
<tr>
<td>500</td>
<td>14.5</td>
<td>7.6</td>
<td>4.6</td>
</tr>
<tr>
<td>750</td>
<td>12.5</td>
<td>3.4</td>
<td>4.1</td>
</tr>
<tr>
<td>1000</td>
<td>10.4</td>
<td>2.4</td>
<td>5.8</td>
</tr>
<tr>
<td>Σ</td>
<td>25.5</td>
<td>18.2</td>
<td>16.7</td>
</tr>
</tbody>
</table>

Optimized over 6200 interferences, Evaluated over 620,000 interferences
Summary

• We exploit very large scale microphone arrays by choosing optimal subsets of microphones

• Adaptive subset selection per target location is then possible
Questions?

OLUA

worst gain = 10.4 dB
(1000 hz)

Multi-frequency $\lambda$

worst gain = 2.4 dB
(1000 hz)
How to Find Significant Weights
For $\lambda$ fixed, significant weights found by thresholding.
Desire 32 microphones (significant weights)
1ˢᵗ microphone weight at least 1000x larger than 3³ʳᵈ microphone weight
Significant Microphones

Insignificant Microphones

\[ \sum f_r \]

\[ |W_r| \]

Sorted Filter Weight Index

\[ 10^0 \to 10^{-10} \]
Simulated Annealing
Simulated Annealing

worst "broadband" gain

microphone configuration index
large search space (1000 chose 30 ≈ \(10^{57}\)) ...
large search space (1000 chose 30 \approx 10^{57}) ...

randomly perturb microphone configuration for next candidate!
many **local minima**, easy for search to get stuck ...
many local minima, easy for search to get stuck ...
decrease probability of accepting worse solution over time
decrease probability of accepting worse solution over time

hope solution settles to local minimum
Simulated Annealing: Global Random Search

1) Large search space, randomly perturb to find next candidate point.
2) If candidate provides better cost, accept. If candidate provides worst cost, accept with some probability.
3) Decrease probability of accepting worse solutions over time.
Optimization Background
1996 (Boyd): finding optimal filter weights convex optimization problem!
Find “\(\mathbf{w}\)” that minimizes \(\|\mathbf{H}\mathbf{w}\|_\infty\)
subject to \(\mathbf{T}^*\mathbf{w} = 1\)
2013 (YL, RB, HC, JR) add l1 penalty for sparse solution, multi-frequency
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\[ w_5 \sim 0 \]

\[ w_6 \text{ vector, weight for each frequency} \sim 0 \]
Example: $N=6$, $F=2$ (two frequencies of interest)

\[
\begin{bmatrix}
w_1(f_1) & w_1(f_2) \\
\end{bmatrix}
\begin{bmatrix}
w_2(f_1) & w_2(f_2) \\
\end{bmatrix}
\begin{bmatrix}
w_6(f_1) & w_6(f_2) \\
\end{bmatrix}
\begin{bmatrix}
w_3(f_1) & w_3(f_2) \\
\end{bmatrix}
\begin{bmatrix}
w_5(f_1) & w_5(f_2) \\
\end{bmatrix}
\begin{bmatrix}
w_4(f_1) & w_4(f_2) \\
\end{bmatrix}
\]
How to Calculate Gain
calculate gain for source $I_0$ at frequency $f_0$
signal received at sensor 2 from source $l_0$

$H_{ln}$ models propagation

$$\tilde{X}_{l_02}(f) = X_{l_0}(f)H_{l_02}(f)$$
direct path model

\[ H_{ln}(f) = \frac{1}{d(l,n)} e^{-j2\pi f \cdot c_{\text{sound}} \cdot d(l,n)} \]
each sensor “n”, for frequency “f”, implements gain $\alpha_f$ and delay $\Delta_f$

**delay** in time domain

$\xrightarrow{\text{Fourier}}$

multiply by complex exponential in frequency domain

$$w_n(f) = \alpha_f e^{-j2\pi f \cdot \Delta_f} = a_f + j b_f$$
References


