A level set method for fluid displacement in realistic porous media

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Outline

- Introduction
- Modeling
  - Level Set Method
  - PQS Algorithm (Prodanović/Bryant)
  - Contact angle modeling
- Results
- Conclusions
Pore scale immiscible fluid displacement

- Fluid-fluid interface (meniscus) at equilibrium with constant capillary pressure $P_c$ and interfacial tension $\sigma$ satisfies Young-Laplace equation

$$P_c = P_{nw} - P_w = \sigma \kappa$$

- We assume quasi-static displacement and thus at each stage interfaces are constant mean curvature ($\kappa$) surfaces

Fig.1. Contact angle at equilibrium satisfies

$$\sigma_{AB} \cos \theta = \sigma_{SA} - \sigma_{SB}$$
Statement of the problem

- **Goal**
  - Accurately model capillarity dominated fluid displacement in porous media

- **What is the big deal?**
  - Calculating constant curvature surfaces (in irregular pore space)
  - Accounting for the splitting and merging of the interface within the pore space

- **What do we do?**
  - Adapt the level set method for quasi-static fluid displacement
Introduction

Modeling
- Level Set Method
- PQS Algorithm (Prodanović/Bryant)
- Coupling with Sediment Mechanics

Results

Conclusions
Level Set Method

- Osher & Sethian, ’88.
- The moving interface is imbedded as the zero level set of function $\phi$; the governing PDE is
  \[ \phi_t + F |\nabla \phi| = 0, \quad \text{given} \quad \phi(\vec{x}, 0) \]

- $F$ is particle speed in the normal direction
- The method works in any dimension and handles topological changes naturally
- Sample speed $F$
  \[ F(x, t) = a_0 - b_0 \kappa(x, t) \]
Progressive quasi-static algorithm

- **Drainage** starts with a planar front exposed to a **slightly compressible curvature model** until steady state is reached:
  \[ F(\vec{x}, t) = a_0 \exp[f(1 - \frac{V(t)}{V_m})] - b_0 \kappa(\vec{x}, t) \]

- Further, we increment curvature and run a **prescribed curvature model**
  \[ F(x, t) = a_0 - b_0 \kappa(x, t) = a_0 - b_0 \nabla \cdot \frac{\nabla \phi}{|\nabla \phi|} \]

- **Imbibition** starts from drainage endpoint and decrements curvature

- Zero contact angle: **wall BC** \( \phi(\vec{x}, t) \leq \psi(\vec{x}) \)

- **Contact angle model**
  \[ F(\vec{x}, t) = aH(-\psi) - \nabla \cdot (b(\psi) \frac{\nabla \phi}{|\nabla \phi|}) \]

  \[ b(\psi) = \begin{cases} 
  |\sigma_{SA} - \sigma_{SB}|, & \text{if } \psi \geq 0 \\
  \sigma_{AB}, & \text{if } \psi < 0.
  \end{cases} \]

\[ \sigma_{AB} \cos \theta = |\sigma_{SA} - \sigma_{SB}| \]
Motivation & Introduction

Modeling
- Level Set Method
- PQS Algorithm (Prodanović/Bryant)
- Coupling with Sediment Mechanics

Results
- Zero contact angle
- Non-zero contact angle

Conclusions
2D Drainage and Imbibition

Simulation steps (alternating red and green colors) in drainage (controlled by throats) and imbibition (controlled by pores). All computed within 2% rel.abs.err.
Fractured Berea Sandstone

Fracture image courtesy of Dr. Zuleima Karpyn, PSU

Comparison with Experiment

Aperture field

Experiment, $S_w=0.35$

Simulation $S_w=0.28$

asperities

oil
Drainage in Fractured Berea

interfac. tension oil-water
σ=41.2 mN/m

P_2=157 Pa
P_{13}=607 Pa
W fluid
Imbibition in Fractured Berea

- $P_9 = 813\ \text{Pa}$
- $P_{25} = 153\ \text{Pa}$
- $P_{26} = 113\ \text{Pa}$
- $P_{28} = 30\ \text{Pa}$
Pc-Sw curve for Fractured Berea
Fractured Sphere Pack

Pore-grain surface sphere radii $R=1.0$
Image size $160^3$ ($dx=0.1$)

NW phase surface in fracture (drainage beginning)
Drainage and Imbibition

Drainage, \( C=4.9 \)

Imbibition, \( C=0.24 \)

Imbibition – rotated
\( C=2.15 \)

Trapped NW phase

Drainage movie

Imbibition movie
Curvature – saturation curves
Throat in 2D: $\theta=60$

Some overlap with solid allowed in order to form contact angle

The last stable meniscus shown in purple: not at geometrical throat!
Fracture in 2D: $\theta=30$

Drainage

Imbibition

- LSMPQS steps shown in alternate red and green colors
Fracture in 2D: $\theta=80$

- Drainage
- Imbibition: does not imbibe at a positive curvature!

- LSMPQS steps shown in alternate red and green colors
Fracture 2D: drainage curves
Fracture in 2D: imbibition curves
Fractional wettability: $\theta = 10$ and 80

Simulation: $C = 4.16$

Analytic solution: 4.23

- Last stable meniscus shown in purple
Mixed wettability: $\theta=60$ and 30

$C=5.73$
Throat3D: $\theta=30$

- Throat is bounded by 4 rods in rhomboidal arrangement
- Note: Movie (click button!) shows only non-wet phase surface colored red (meniscus) and gray (solid contact)

C=7.5, last stable main meniscus

C=7.6, only pendular rings remain
Conclusions

- Drainage/imbibition modeling is
  - Geometrically correct
  - Robust with respect to geometry
- We identify (independently confirm)
  - Haines jumps at drainage
  - Melrose criterion at imbibition
- We can easily obtain Pc-Sw curves, fluid configuration details (volumes, areas)
- Capillarity has an important effect on flow in rough wall fractures with contact points – we find W phase blobs around contacts and hysteresis of C-Sw curves
- Modeling (fractional & mixed) wettability possible
Thank you!

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