The class covers fundamentals of convergence analysis for Finite Element Method and linear boundary-value problems with a focus on a different set of problems changing every semester. So, in principle, the class may be taken multiple times.

This semester we will cover the fundamentals of the convergence theory for conforming methods using the energy spaces forming the exact sequence: $H^1, H(\text{curl}), H(\text{div})$ and $L^2$ spaces. The class is recommended for the first time students.

We will start by reviewing fundamentals on variational formulations and energy spaces reviewing classical model problems: diffusion-convection-reaction, linear acoustics, Maxwell equations, elastodynamics, Stokes equations, coupled problems. Most of the material will be presented w/o proofs but with illustrative examples.

We will continue then with the classical results on abstract variational problems and corresponding fundamental stability and convergence results for the Galerkin method: Ritz theory, Cea’s Lemma, Babuška’s Theorem, Brezzi’s theory for mixed problems, indefinite problems and concept of asymptotic convergence, Petrov-Galerkin method with optimal test functions.

We shall focus then on the exact sequence structure and known polynomial exact sequences corresponding to classical continuous, Nédélec and Raviart-Thomas finite elements, followed by the development of classical abstract interpolation theory and the $h$-convergence analysis for conforming methods.

The end of the semester will be devoted to more advanced topics: a-priori error estimation for $h$-adaptive methods, discussion on $p$- and $hp$-versions of finite elements.

I will use my 1D and 2D $hp$ codes to illustrate the theoretical estimates.

CLASS OUTLINE SCHEDULE:

**Jan 17 - Jan 20** Examples of boundary-value and initial boundary-value problems and variational formulations.

**Jan 23 - Jan 27** Well-posedness of a variational problem, Babuška - Nečas Theorem.

**Jan 30 - Feb 3** Standard energy spaces: $H^1, H(\text{curl}), H(\text{div}), L^2$ and the exact sequence.

**Feb 6 - Feb 10** Abstract variational problem. Galerkin and Ritz methods.

**Feb 13 - Feb 17** Coercive problems: Lax-Milgram Theorem and Cea’s Lemma.

**Feb 20 - Feb 24** Babuška’s Theorem and notion of discrete stability.

**Feb 27 - Mar 3** Mikhlin’s theory and concept of asymptotic stability.
Mar 6 - Mar 10  Brezzi's theory and mixed problems.

Mar 13 - Mar 17  Spring break.

Mar 20 - Mar 24  Construction of $H^1$-, $H$(curl)-, $H$(div)-, and $L^2$-conforming elements.


Apr 3 - Apr 7  Convergence in weaker norms. Aubin-Nitsche trick.

Apr 10 - Apr 14  Accounting for integration error. First Strang's lemma.

Apr 17 - Apr 21  Convergence analysis for $h$-adaptive methods.

Apr 24 - Apr 28  Conforming $p$ and $hp$ finite elements. Projection-based interpolation theory.

May 1 - May 5  Review.

The class will be conducted in a seminar style. No exams (including final) will be given. Instead, problems with varying difficulty, ranging from ”hard theory”, through practice exercises, to assignments involving numerical experiments , will be assigned in the class on a continuous basis. Each problem will be worth a number of points (5-20). The final grade will be determined by the number of collected points.

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<th>Final score range</th>
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<td>100 and above</td>
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Discussion session; one and a half hour, once a week, to be determined (most likely Fri, 3:00 - 4:30).

Instructor: Dr. Leszek Demkowicz, POB 6.326, Office hours: immediately after the class.