Exam 1:

1. Principle of mathematical induction
2. Bijection between the family of all equivalence classes and the family of all partitions for a set
3. Properties of direct and inverse image
4. Characterization of a bijection
5. Comparability of cardinal numbers
6. Properties of open sets in $\mathbb{R}^n$
7. Properties of closed sets in $\mathbb{R}^n$
8. Relation between open and closed sets (duality principle)
9. Properties of the interior operation
10. Properties of the closure operation

Exam 2:

1. Characterization of accumulation (limit) points with sequences
2. Equivalence of continuity and sequential continuity in $\mathbb{R}^n$
3. The Bolzano-Weiestrass Theorem for Sets
4. The Bolzano-Weiestrass Theorem for Sequences
5. The Weierstrass Theorem
6. Characterization of \( \lim \inf \)
7. Characterization of a direct sum of two vector subspaces
8. Characterization of a Hamel basis
9. Existence of a Hamel basis in a vector space (w/o proof)
10. Rank and Nullity Theorem
11. Characterization of a projection
12. Construction of a dual basis in a finite-dimensional space,
13. Properties of orthogonal complements
14. Properties of transpose operators
15. Relation between ranks of a linear map and the rank of its transpose
16. Cauchy-Schwartz Inequality
17. Properties of adjoint operators

**Exam 3:**

1. Properties of a \( \sigma \)-algebra (Prop. 3.1.1)
2. Properties of an (abstract) measure (Prop. 3.1.6)
3. Properties of Borel sets (Prop. 3.1.4, 3.1.5 combined)
4. Characterization of Lebesgue measurable sets (Prop. 3.2.3, Thm 3.2.1)
5. Cartesian product of Lebesgue measurable sets (Thm. 3.2.2)
6. Properties of measurable (Borel) functions (Prop. 3.4.1)
7. Properties of Lebesgue integral (Prop. 3.5.1)
8. Fatou’s Lemma

9. Lebesgue Dominated Convergence Theorem (for non-negative functions, Thm. 3.5.2)

10. Hölder and Minkowski inequalities,

11. Properties of open sets, properties of closed sets, properties of the operations of interior and closure (all in context of general topological spaces),

12. Characterization of open and closed sets in a topological subspace.

**Additional material for the final:**

1. Characterization of (globally) continuous functions (Prop. 4.3.2),

2. Properties of compact sets,

3. The Heine-Borel Theorem,

4. The Weierstrass Theorem,

5. Properties of sequentially compact sets (Prop. 4.4.5),

6. Hölder and Minkowski inequalities for sequences,

7. Completness of Chebyshev, $l^p$, and $L^p$ spaces,

8. Bolzano-Weiestrass Theorem,