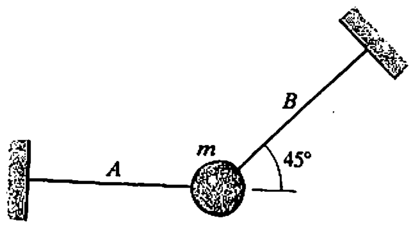
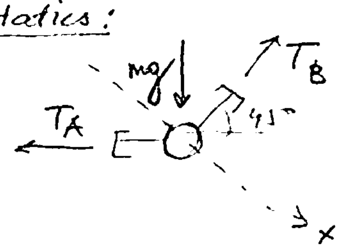
EM311M - Dynamics
Exam 2
Wednesday, Oct 28, 2009, 6:00 - 8:00 p.m., BEL 328

Solutions

1. The ball of mass m is suspended by cables A and B. Cable B is cut. Is the force in cable A going to increase or decrease ? Explain. (5 points)



Statics:

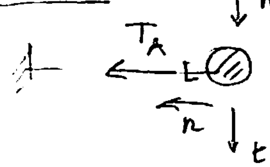


$\Sigma F_x = 0$

$-T_A \cos 45° + mg \cos 45° = 0$

$\therefore \boxed{T_A = mg}$

Dynamics:



$m a_t = mg$

$m a_n = T_A$ But $a_n = \frac{v^2}{r} = 0$ (starts from rest)

so $\boxed{T_A = 0}$

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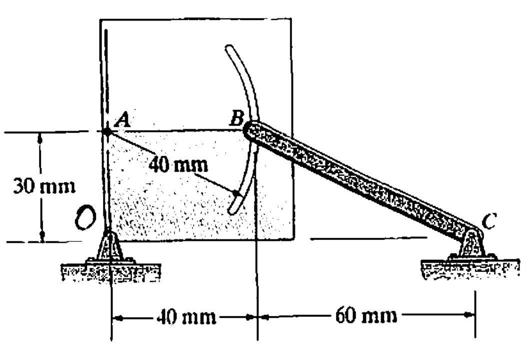
The force will decrease!

2. Bar BC rotates with a constant counterclockwise angular velocity of 2 rad/s. Determine the angular velocity of the plate. (5 points)



$$\underset{\sim}{v}_B = \cancel{v_C}^{0} + \underset{\sim}{\omega}_{BC}\,(0,\ 0,\ 2) \times \underset{\sim}{CB}\,(-0.06,\ 0.03,\ 0)$$
$$(-0.06,\ -0.12,\ 0)$$ (2)

$$\underset{\sim}{v}_B = \underset{\sim}{v}_O^{\,0} + \underset{\sim}{\omega}_P\,(0,\ 0,\ \omega_p) \times \underset{\sim}{OB}\,(0.04,\ 0.03,\ 0)$$
$$(-0.03\omega_p,\ 0.04\omega_p,\ 0)$$
$$+\ \underset{\sim}{v}_{B/rel}\,(0,\ v_{B/rel},\ 0)$$
$$= (-0.03\omega_p,\ 0.04\omega_p + v_{B/rel},\ 0)$$ (2)

Matching components:

$-0.06 = -0.03\omega_p \Rightarrow \underline{\omega_p = 2\ [\frac{rad}{s}]}$ (1)

$-0.12 = 0.04\omega_p + v_{B/rel} = 0.08 + v_{B/rel} \Rightarrow v_{B/rel} = -0.20\ [\frac{m}{s}]$

3. Derive the principle of work and energy for a single particle. (5 points)

$$a_t\,ds = v\,dv$$

$$m a_t\,ds = m v\,dv$$

$$\underset{\sim}{F}\cdot d\underset{\sim}{r} = F_t\,ds = m v\,dv$$

$$\int_{s_1}^{s_2} \underset{\sim}{F}\cdot d\underset{\sim}{r} = \int_{v_1}^{v_2} m v\,dv = \frac{1}{2} m v^2\Big|_{v_1}^{v_2} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

So: $\boxed{\frac{1}{2} m v_1^2 + \int \underset{\sim}{F}\cdot d\underset{\sim}{r} = \frac{1}{2} m v_2^2}$

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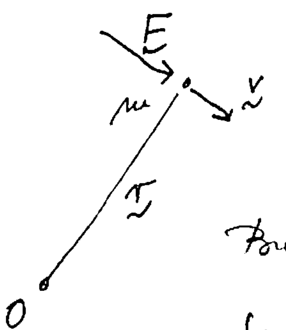
4. Derive the principle of angular impulse and momentum for a single particle. (5 points)



$$m\dot{\underset{\sim}{v}} = m\underset{\sim}{a} = \underset{\sim}{F} \quad / \ \underset{\sim}{r}\times$$

$$\underset{\sim}{r}\times m\dot{\underset{\sim}{v}} = \underset{\sim}{r}\times\underset{\sim}{F}$$

but $(\underset{\sim}{r}\times m\underset{\sim}{v})^{\cdot} = \overset{0}{\cancel{\underbrace{\dot{\underset{\sim}{r}}}_{\underset{\sim}{v}}\times m\underset{\sim}{v}}} + \underset{\sim}{r}\times m\dot{\underset{\sim}{v}}$

so $(\underbrace{\underset{\sim}{r}\times m\underset{\sim}{v}}_{\underset{\sim}{H}_O})^{\cdot} = \underset{\sim}{r}\times\underset{\sim}{F} = \underset{\sim}{M}_O \quad / \int_{t_1}^{t_2} dt$

$$\boxed{\underset{\sim}{H}_O(t_2) - \underset{\sim}{H}_O(t_1) = \int_{t_1}^{t_2} \underset{\sim}{M}_O\,dt}$$

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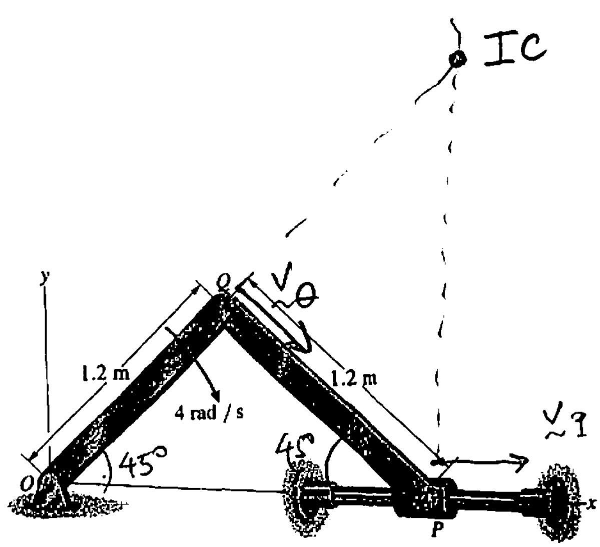
5. Determine (just point to) the instanteneous center of zero velocity of member QP. (5 points)



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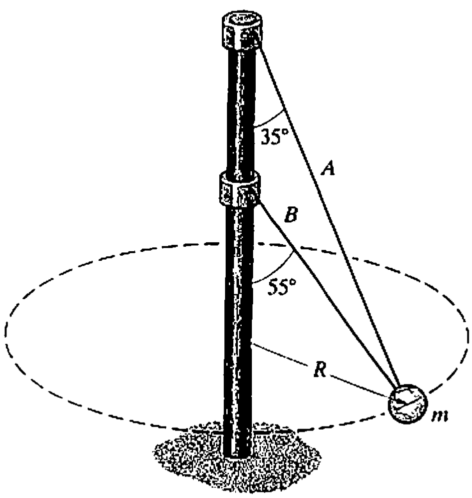
6. The 10-kg mass m rotates around the vertical pole in a horizontal circular path of radius $R = 1\text{m}$. For what range of values of the velocity v of the mass will the mass remain in the circular path described? (25 points)



In the extreme scenarios (for min or max velocity), one of the cables goes slack.

So, we need to consider two scenarios.

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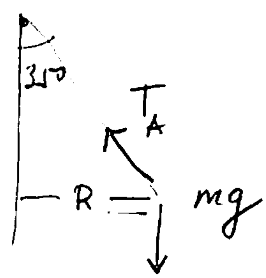
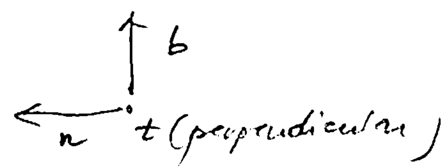
Case 1. B goes slack





$m a_t = 0 \Rightarrow a_t = 0 \Rightarrow V = \text{const}$

$m \cancel{a_b}^{\,0} = T_A \cos 35^\circ - mg \Rightarrow T_A = \dfrac{mg}{\cos 35^\circ}$

$m \underset{\frac{v^2}{R}}{\overset{}{a_n}} = T_A \sin 35^\circ \Rightarrow \dfrac{\cancel{m} v^2}{R} = \dfrac{mg}{\cos 35^\circ} \sin 35^\circ = \cancel{m}g \tan 35^\circ$

$v^2 = Rg \tan 35^\circ = 1 \cdot 9.81 \cdot \tan 35^\circ$

$V = 2.62 \left[\frac{m}{s}\right]$

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Case 2. A goes slack

Same free body diagram and algebra with 55° replacing 35°

$v^2 = Rg \tan 55^\circ = 1 \cdot 9.81 \cdot \tan 55^\circ$

$V = 3.74 \left[\frac{m}{s}\right]$

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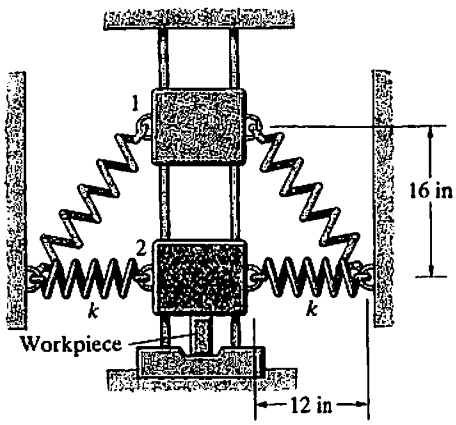
Velocity will range between 2.62 and 3.74 $\frac{m}{s}$

7. The 100-lb weight is released from rest in position 1. The spring constant is $k = 120$ lb/ft, and the springs are unstretched in position 2. If the coefficient of restitution of the impact of the weight with the workpiece in position 2 is $e = 0.6$, what is the magnitude of the velocity of the weight immediately after the impact ? (25 points)



Step 1: Principle of Work and Energy

$$\cancel{T_1}^{\,0} + U_{12} = T_2$$

Working forces ; weight, two spring forces

$$U_{12} = 100\ \text{lb} \cdot \frac{16}{12} + 2\left(\frac{1}{2} \cdot 120\left(\left(\frac{2}{3}\right)^2 - 0\right)\right)$$

$$= 133.33 + 53.33 = 186.67\ [\text{lb} \cdot \text{ft}]$$

stretch in position 1

$$\Delta L = \sqrt{12^2 + 16^2} - 12 = 8\ \text{in} = \frac{2}{3}\ \text{ft}$$

Assume no friction

$$186.67 = \frac{1}{2}\,\frac{100}{32.2}\,V_2^2$$

$$\therefore\ V_2 = 10.96\ \left[\frac{\text{ft}}{\text{s}}\right]$$

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Step 2: Impact

velocity of the weight before $V_w^{before} = 10.96\ \left[\frac{\text{ft}}{\text{s}}\right]$

$$e = \frac{V_w^{after} - \cancel{V_{workpiece}^{after}}^{\,0}}{\cancel{V_{workpiece}^{before}}^{\,0} - V_w^{before}}$$

$$\therefore\ V_w^{after} = -e\,V_w^{before} = -6.58\ \left[\frac{\text{ft}}{\text{s}}\right]$$

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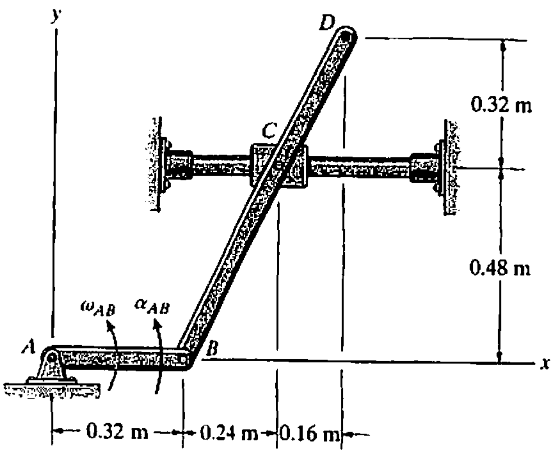
8. The angular velocity and angular acceleration of bar AB are $\omega_{AB} = 2$ rad/s and $\alpha_{AB} = 8$ rad/s². What is the acceleration of point D ? (25 points)



Velocities

$$\underset{\sim}{v}_B = \cancel{\underset{\sim}{v}_A}^{0} + \begin{array}{r} \omega_{AB}\,(0,\ 0,\ 2) \\ \times\ \underset{\sim}{AB}\,(0.32,\ 0,\ 0) \\ \hline (0,\ 0.64,\ 0) \end{array}$$

$$\underset{\sim}{v}_B = \underset{\sim}{v}_C + \underset{\sim}{\omega}_{BD} \times \underset{\sim}{CB}$$

$$= (v_{Cx},\ 0,\ 0) + \begin{array}{r} \underset{\sim}{\omega}_{BD}\,(0,\ 0,\ \omega_{BD}) \\ \times\ \underset{\sim}{CB}\,(-0.24,\ -0.48,\ 0) \\ \hline (0.48\,\omega_{BD},\ -0.24\,\omega_{BD},\ 0) \end{array}$$

Matching components :

$$0 = v_{Cx} + 0.48\,\omega_{BD}$$

$$0.64 = -0.24\,\omega_{BD} \qquad \Rightarrow \omega_{BD} = -2.67\left[\tfrac{\text{rad}}{\text{s}}\right]$$

$$\Rightarrow v_{Cx} = -0.48 \cdot \omega_{BD} = 1.28\left[\tfrac{\text{m}}{\text{s}}\right]$$

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Accelerations :

$$\underset{\sim}{a}_B = \cancel{\underset{\sim}{a}_A}^{0} + \begin{array}{r} \underset{\sim}{\alpha}_{AB}\,(0,\ 0,\ 8) \\ \times\ \underset{\sim}{AB}\,(0.32,\ 0,\ 0) \\ \hline (0,\ 2.56,\ 0) \end{array} - \underbrace{\omega_{AB}^2}_{4} \cdot \underset{\sim}{AB}\,(0.32,\ 0,\ 0)$$

$$= (-1.28,\ 2.56,\ 0)\ \left[\tfrac{\text{m}}{\text{s}^2}\right]$$

$$\underset{\sim}{a}_B = \underset{\sim}{a}_C(a_{Cx},\ 0,\ 0) + \begin{array}{r} \underset{\sim}{\alpha}_{BD}\,(0,\ 0,\ \alpha_{BD}) \\ \times\ \underset{\sim}{CB}\,(-0.24,\ -0.48,\ 0) \\ \hline (0.48\,\alpha_{BD},\ -0.24\,\alpha_{BD},\ 0) \end{array} - \omega_{BD}^2 \cdot \underset{\sim}{CB}\,(-0.24,\ -0.48,\ 0)$$

$$= (a_{Cx} + 0.48\,\alpha_{BD} + 1.71,\ -0.24\,\alpha_{BD} + 3.42,\ 0)$$

Matching y components $\Rightarrow$

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$$2.56 = -0.24\,\alpha_{BD} + 3.42 \quad \Rightarrow \alpha_{BD} = 3.583\ \left[\tfrac{\text{rad}}{\text{s}^2}\right]$$

$$\underset{\sim}{a}_D = \underset{\sim}{a}_B(-1.28,\ 2.56,\ 0) + \begin{array}{r} \underset{\sim}{\alpha}_{BD}\,(0,\ 0,\ 3.583) \\ \times\ \underset{\sim}{BD}\,(0.4,\ 0.8,\ 0) \\ \hline (\overset{5}{-2.86},\ 1.43,\ 0) \end{array} - \underbrace{\omega_{BD}^2}_{2.67^2}\,\underset{\sim}{BD}\,(0.4,\ 0.8,\ 0)$$

$$= (-6.99,\ -1.41,\ 0)\ \left[\tfrac{\text{m}}{\text{s}^2}\right]$$

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