1. Define (discuss) the following notions (5 points each).

- entropy,
- the grad-curl-div exact sequence,
- Legendre polynomials and Legendre functions,
- generalized statistical interpretation of a wave function.
- Legendre transform.

2. Recall the Coulomb’s law and compute the total force exerted on point charge \( q \) by a line charge distribution with a constant charge density \( q_1 \) (see Fig. 1). (25 points).

![Figure 1: A line with charge.](image)

3. Consider the 1D time-independent Schrödinger equation with the harmonic potential \( V = \frac{1}{2}\omega^2 x^2 \), and reproduce the computation of bound states leading to Hermite polynomials (25 points).

4. Consider a system consisting of two particles in a plane: a particle with mass \( 2m \) that must stay on a circle with radius \( R \), and a particle of mass \( m \), connected to the first particle with a rigid link of length \( l \) (see Fig. 2). The particles are in the gravitational field of the Earth (potential energy \( V = mgy \)). Introduce appropriate generalized coordinates, compute Lagrangian and write down the Lagrange
equations of motion for the system. Use Legendre transform to compute the Hamiltonian and write down the corresponding Hamilton equations (25 points).