Yagi-Uda Antenna

by

Dong Xue

Department of Engineering Mechanics

May 5th, 2002

Contents

1 Introduction **4**

2 Motivation and objectives **5**

3 Problem setup and analysis **6**

3.1 Formulations 6

3.2 NBS design 8

4 Matlab implementation **10**

4.1 Structure of the code 10

4.2 Highhight of the code 11

5 Simulation results **12**

5.1 E-plane 12

5.2 H-plane 14

5.3 Complex current distributions 15

5.4 Characteristic variables 30

6 Conclusion **31**

A List of Routine **32**

A.1 yagi.m 32

A.2 func.m 32

A.3 func2.m 32

List of Figures

1.1 Geometry of Yagi-Uda array 4

3.1 Configure of 15 element NBS Yagi antenna 9

5.1 The E-plane 13
5.2 The H-plane 14
5.3 The current distribution on the reflector N=14 15
5.4 The current distribution on the feeder N=15 16
5.5 The current distribution on the director N=1 17
5.6 The current distribution on the director N=2 18
5.7 The current distribution on the director N=3 19
5.8 The current distribution on the director N=4 20
5.9 The current distribution on the director N=5 21
5.10 The current distribution on the director N=6 22
5.11 The current distribution on the director N=7 23
5.12 The current distribution on the director N=8 24
5.13 The current distribution on the director N=9 25
5.14 The current distribution on the director N=10 26
5.15 The current distribution on the director N=11 27
5.16 The current distribution on the director N=12 28
5.17 The current distribution on the director N=13 29

Chapter 1

Introduction

Yagi-Uda Antenna is a parasitic linear array of parallel dipoles, see Fig 1.1, one of which is energized directly by a feed transmission line while the other act as parasitic radiator whose currents are induced by mutual coupling. The basic antenna is composed of one reflector (in the rear), one driven element, and one or more directors (in the direction of transmission/reception).The Yagi-Uda antenna has received exhaustive analytical and experimental investigations in the open literature and else where. The characteristics of a Yagi are affected by all of the geometric parameters of the array. Usually Yagi-Uda arrays have low input impedance and relatively narrow bandwidth.Improvements in both can achieved at the expense of others. Usually a compromise is made, and it depends on the particular design.

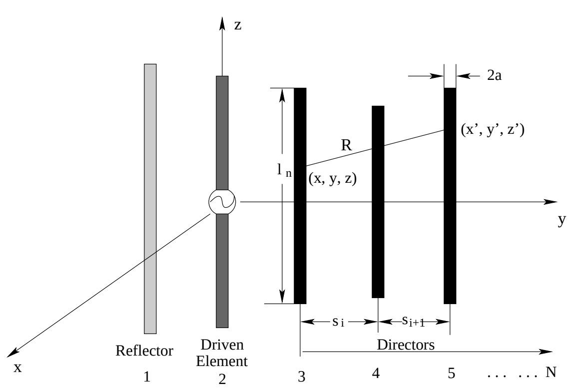

Figure 1.1: Geometry of Yagi-Uda array

Chapter 2

Motivation and objectives

Often one needs to improve reception of a particular radio or television station. One effective way to do this is to build a Yagi-Uda (or Yagi) antenna because of their simplicity and relatively high gain.

The goal of the project *is to simulate an NBS yagi antenna which covers all the VHF TV channels.* we will calculate and plot

- Far-zone field (both E-plane and H-plane)
- 3-db beamwidths
- Front-to-back ratio
- Directivity
- Complex current distribution on each element
- Input impedance.

We choose frequency $f_0 = 216MHZ$. Why? Because the VHF TV channel starts with 54MHz(channel 2) and ends with 216MHz (channel 13). Antennas' gains rise slowly up to the design frequency and fall off sharply thereafter [5]. It is therefore easier to make the design frequency a little higher than desired.

Chapter 3

Problem setup and analysis

In this Chapter, we will introduce formulations and parameters used in my simulations.

3.1 Formulations

- Pocklington's Integral Equation

  Pocklington's Integral Equation is used on finite diameter wires. We have the field equation [1]:

  $$\boldsymbol{E} = \boldsymbol{E}^i(\boldsymbol{r}) + \boldsymbol{E}^s(\boldsymbol{r}) \tag{3.1.1}$$

  where $\boldsymbol{E}(\boldsymbol{r})$ is the total electric field. $\boldsymbol{E}_s(\boldsymbol{r})$ is the scattered electric field produced by the induced current current density $\boldsymbol{J}_s$. $\boldsymbol{E}_i(\boldsymbol{r})$ is the incident electric field.

  We get derive *Pocklington's integral equation* as followings,

  $$\int_{-l/2}^{+l/2} \boldsymbol{I}_z(z') \left[(\frac{\partial^2}{\partial z^2} + k^2) \frac{e^{-jkR'}}{4\pi R} \right] dz' = j\omega\epsilon E_z^i \tag{3.1.2}$$

  with $R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$.

- Fourier expansion of the current

  $$I_n(z') = \sum_{m=1}^{M} I_{mn} cos \left[(2m-1) \frac{\pi z'}{l_n} \right]$$

where I_{mn} represents the complex current coefficient of mode m on element n and l_n represents the corresponding length of the n element. Pocklinton's integral (0.02)reduces to

$$\begin{aligned} \textstyle\sum_{m=1}^{M} \quad & I_{mn} \left\{ (-1)^{m+1} \frac{(2m-1)\pi}{l_n} G_2 \left(x, x', y, \frac{y}{z}, \frac{l_n}{2} \right) \right. \\ + \quad & \left[k^2 - \frac{(2m-1)^2 \pi^2}{l_n^2} \right] \\ \times \quad & \left. \int_0^{l_0/2} G_2 \left(x, x', y, \frac{y'}{z_n'} \right) cos \left[\frac{(2m-1)\pi z_n'}{l_n} \right] dz_n' \right\} \\ = \quad & j4\pi\omega\epsilon_0 E_z^t \end{aligned} \tag{3.1.3}$$

where

$$G_2 \left(x, x', y, \frac{y'}{z}, z_n' \right) = \frac{e^{-jkR_-}}{\boldsymbol{R}_-} + \frac{e^{-jkR_+}}{\boldsymbol{R}_+} \tag{3.1.4}$$

and

$$\boldsymbol{R}_\pm = \sqrt{(x-x')^2 + (y-y')^2 + a^2 + (z \pm z')^2} \tag{3.1.5}$$

where $n = 1, 2, 3, \ldots, N$ and N = total number of elements. $R_\pm$ is the distance from the center of each wire radius to the center of any other wire, as shown in Fig. 1.1

- Method of Moments We use Method of Moments to obtain the complex current coeffient I_{mn}.

  1. On driven element

     – The matching is done on the **surface**.
     – $E_z^t = 0$ at $M - 1$ points.
     – The Mth equation on the feed element is

     $$\sum_{m=1}^{M} I_{nm}(z' = 0)|_{n=N} = 1$$

  2. On all other element

     – The matching is done at the **center** of the wire

- $E^t_z(z = z_i) = 0$

- Far-Field Pattern

  Once the current distribution is found, the far-zone field generated by summing the contributions from each.

$$\begin{aligned} E_\theta &= \sum_{n=1}^{N} E_{\theta n} = -j\omega A_\theta \\ A_\theta &= \sum_{n=1}^{N} A_{\theta n} = -\frac{\mu e^{-jkr}}{4\pi r} sin\theta \sum_{n=1}^{N} \{ e^{jk(x_n sin\theta cos\phi + y_n sin\theta sin\phi} \\ \textstyle\sum_{m=1}^{M} & I_{nm} \left[\frac{sin(Z^+)}{Z^+} + \frac{sin(Z^-)}{Z^-} \right] \Big\} \frac{l_n}{2} \end{aligned}$$

  with

$$Z^+ = \left[\frac{(2m-1)\pi}{l_n} + kcos\theta \right] \frac{l_n}{2} \quad (3.1.6)$$

$$Z^- = \left[\frac{(2m-1)\pi}{l_n} - kcos\theta \right] \frac{l_n}{2} \quad (3.1.7)$$

3.2 NBS design

A government document has been published which provides extensive data of experimental in investigations carried out by National Bureau of Standards [6].We can obtain the desired data from the government document, see Fig3.1.

- Element Number $N = 15$.
- Radius of each element $a = 0.0085$
- Directors length $l_1 = l_2 = 0.424, l_3 = 0.420, l_4 = 0.407, l_5 = 0.403, l_6 = 0.398, l_7 = 0.394, l8 - l13 = 0.390$
- Reflector length $l_{14} = 0.475$
- Feeder length $l_{15} = 0.466$
- Space between directors is 0.308.

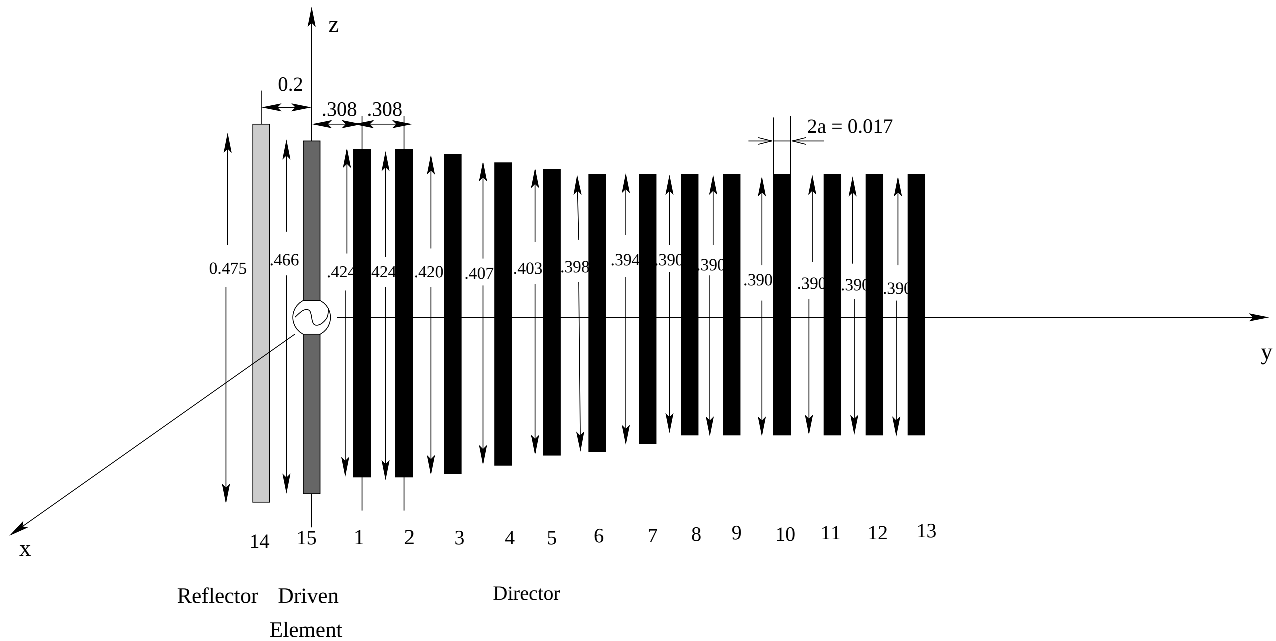

Figure 3.1: Configure of 15 element NBS Yagi antenna

- Space between feeder and reflector is 0.2

The overall antenna length would be $L = 4.2\lambda$. Parameters (element lengths and spacings) are given in terms of *wavelength*.

Chapter 4

Matlab implementation

In this project, I did not use any existing fancy code. Instead, I write some code of my own, using matlab.

4.1 Structure of the code

There are three subroutines of my code. They are The data we should design are: *yagi.m, func.m and func2.m*,See Appendix

In the main subroutine *yagi.m*, the input variables are,

- The total number of elements of the array (N).
- The corresponding lengths of each elements (l_i/λ).
- The spacings between elements (s_{ik}/λ).

The output the the following,

- Far-zone field (both E-plane and H-plane)
- 3-db beamwidths
- Front-to-back ratio
- Directivity
- Complex current distribution on each element
- Input impedance.

4.2 Highhight of the code

In my matlab code, the following points must be emphasized:

- My code is based on pocklington's integral equation. The entire domain cosinusoidal (fourier) basis modes are used for each of the antenna elements..
- All the elements are along the y-axis, with the driven element at the origin.
- If the effect of a mode is found on the element that it is located then distance is the radius a of the element, otherwise the distance is found using the formula $d = \sqrt{(}\rho^2 + a^2)$.
- When we calculate the radiated fields,
  - *E-plane*
$$\phi = \begin{cases} 90^0 & : \quad \theta \in [0^0, 180^0] \\ 270^0 & : \quad \theta \in [180^0, 360^0] \end{cases}$$
  - *H-plane*
$$\theta = 90^0 \quad \phi \in [0^0, 360^0]$$
- When we calculate the directivity
$$\theta = 90^0 \quad \phi = 90^0.$$

Chapter 5

Simulation results

All the following performance figures are theoretical calculations, see Chapter 3. That means, for instance, that the actual gain will be slightly less than that given.

5.1 E-plane

We can plot far-zone Electric Field both in polar coordinate and cartesian coordinate. see Fig.5.1.The corresponding beamwidth and font-to-back ratio are shown in red.

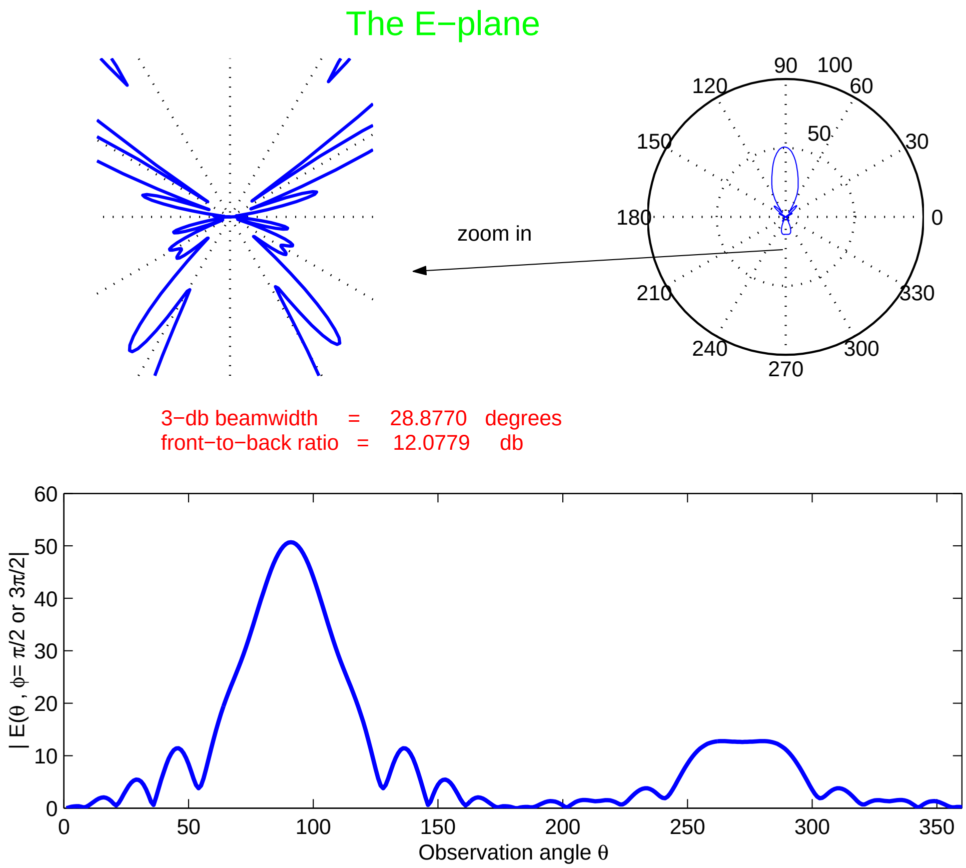

Figure 5.1: The E-plane

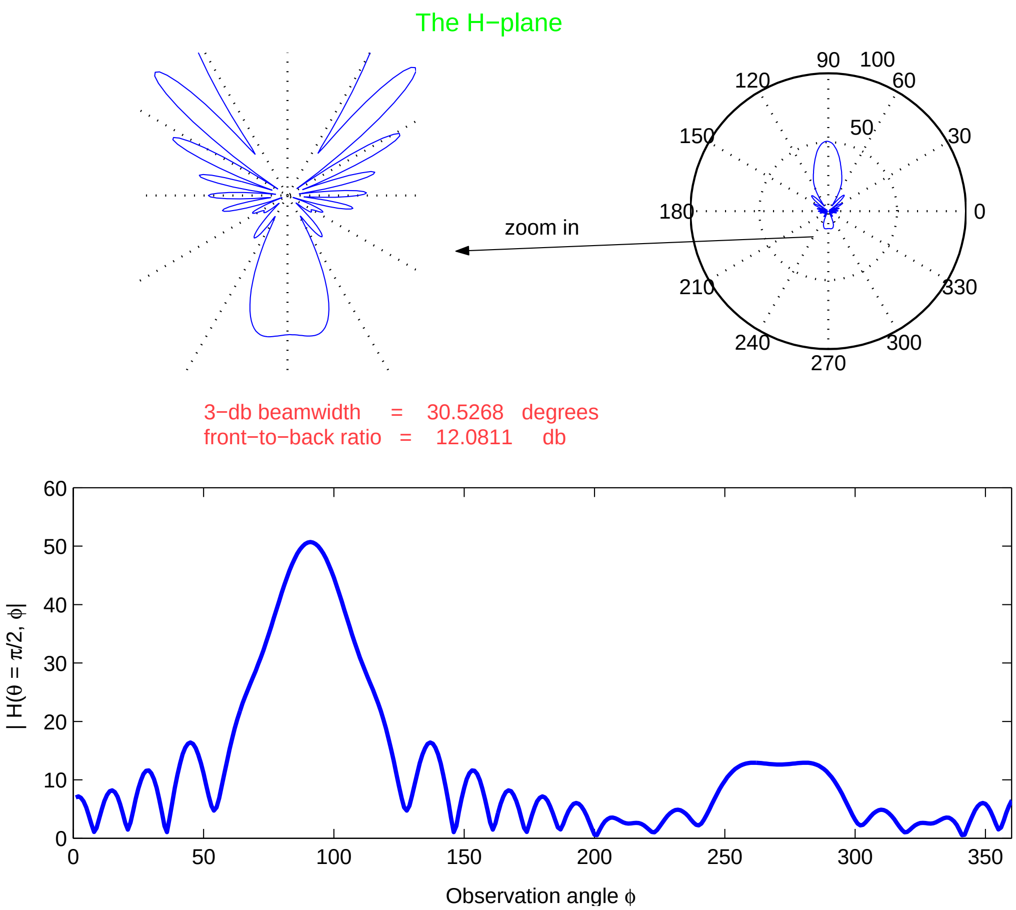

Figure 5.2: The H-plane

5.2 H-plane

We can plot far-zone Magnetic Field both in polar coordinate and cartesian coordinate. see Fig.5.2.The corresponding beamwidth and font-to-back ratio are shown in red.

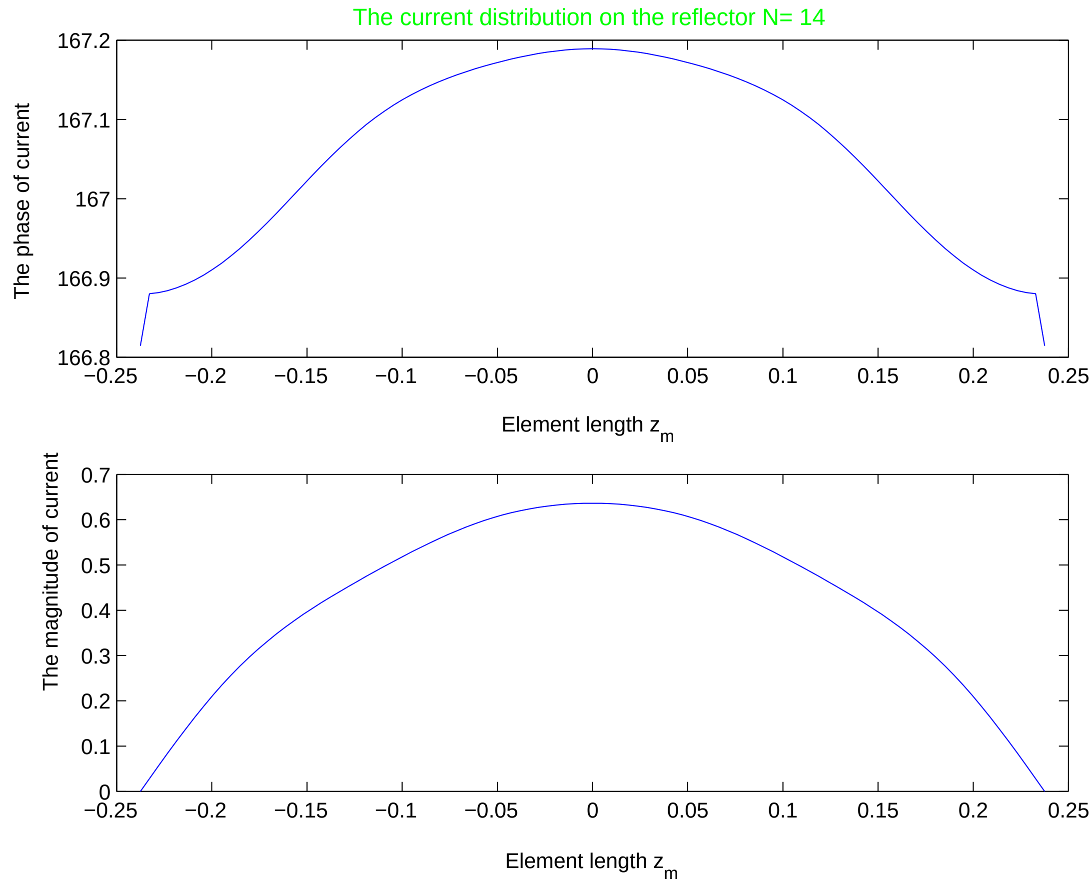

Figure 5.3: The current distribution on the reflector N=14

5.3 Complex current distributions

We can get the complex current distributions(both phase and magnitude) on each of the 15 elements, see the following figures.

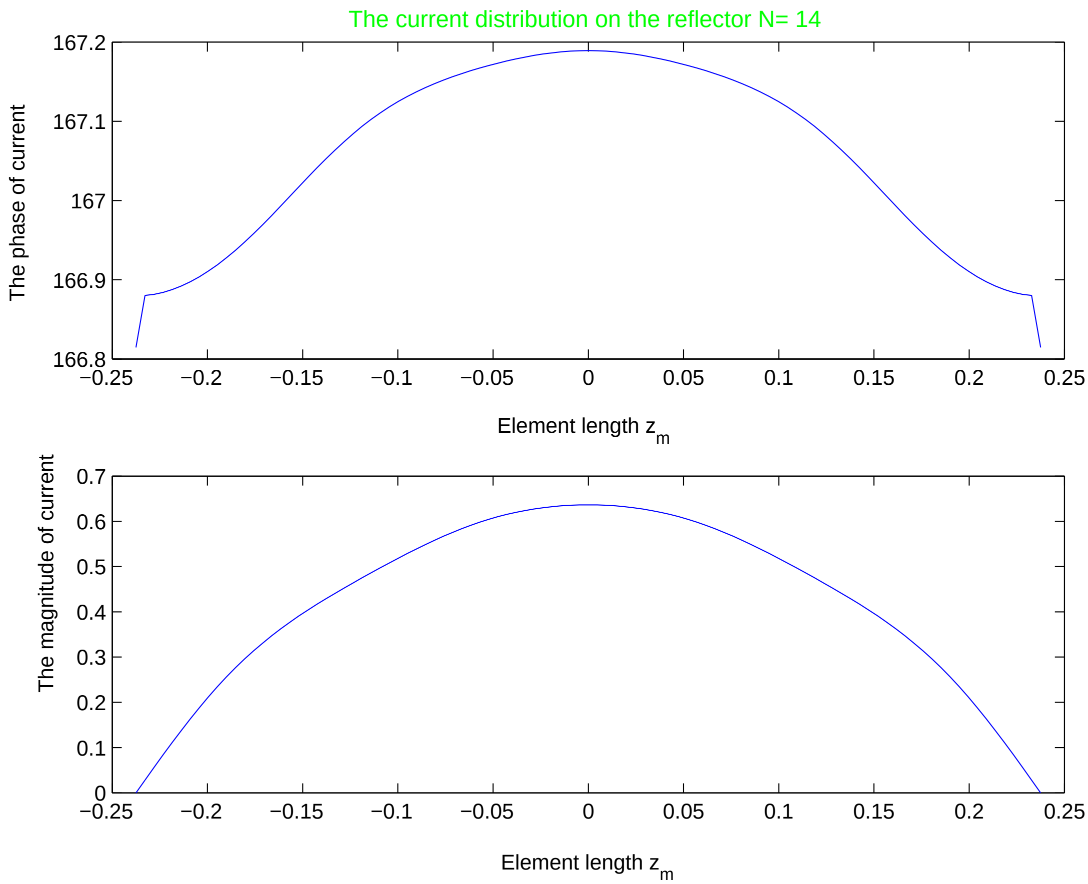

Figure 5.4: The current distribution on the feeder N=15

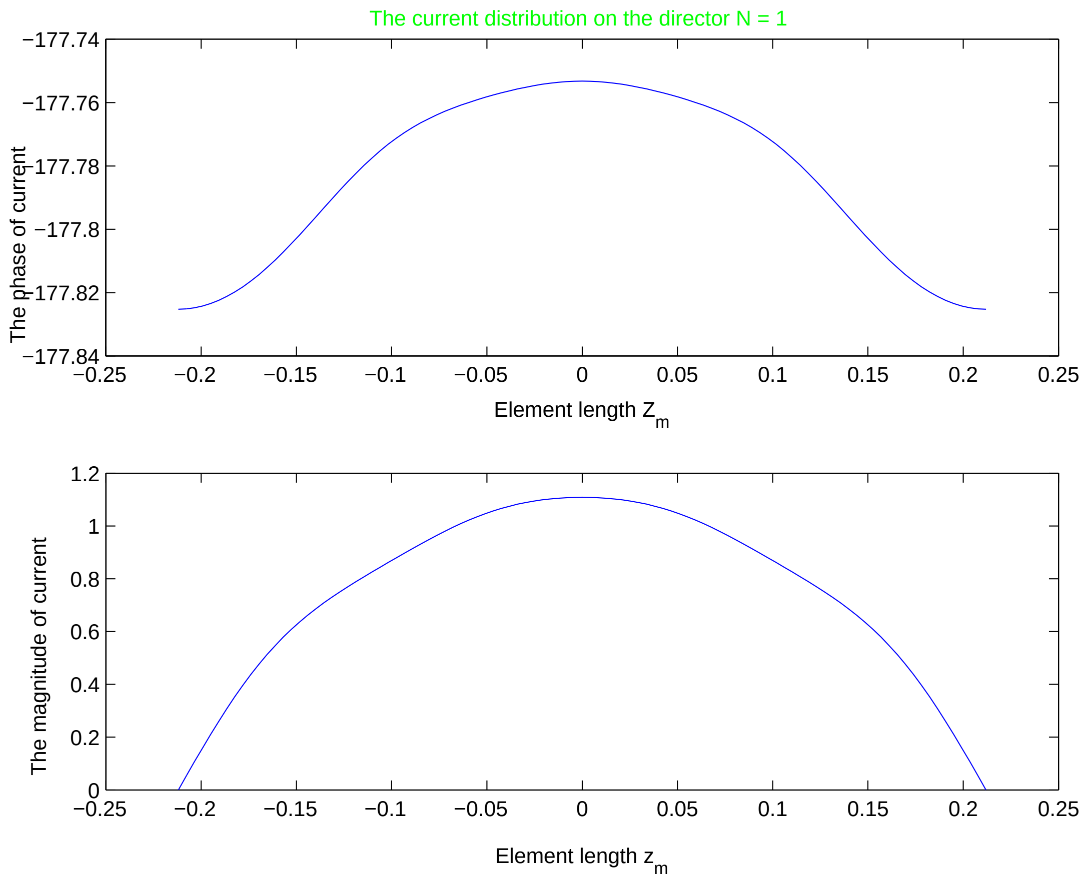

Figure 5.5: The current distribution on the director N=1

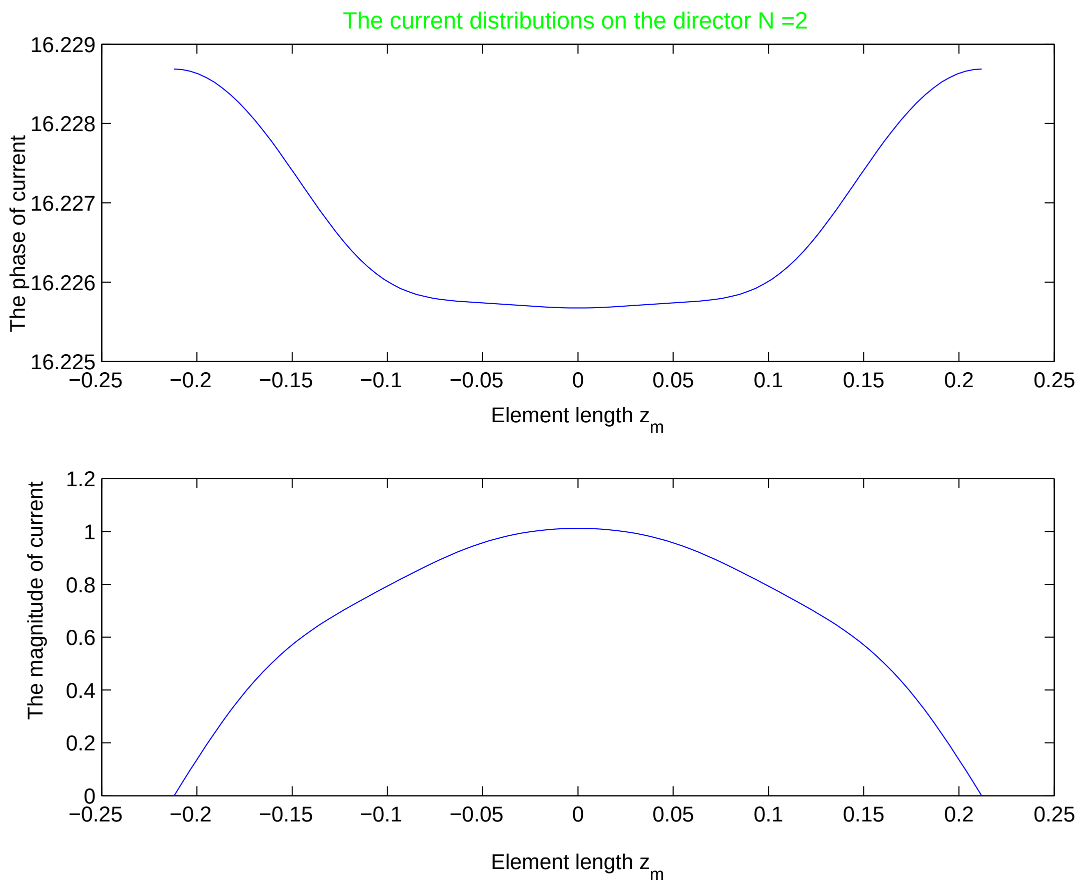

Figure 5.6: The current distribution on the director N=2

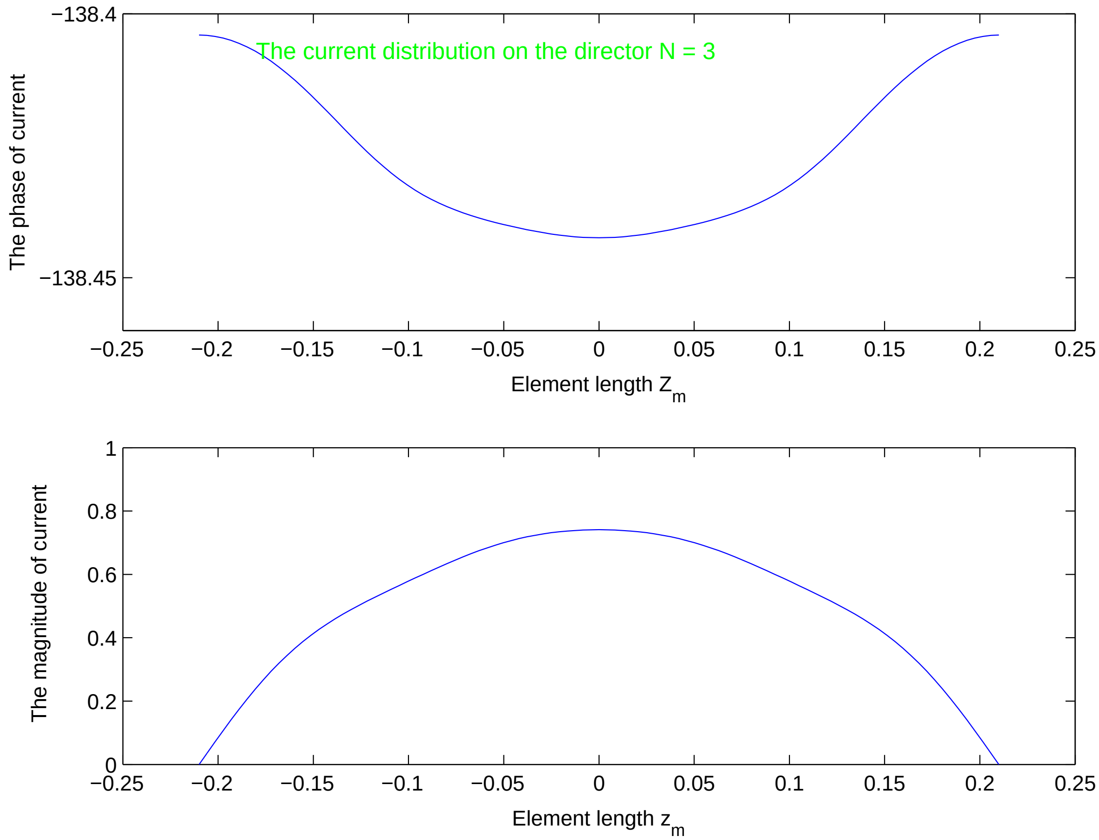

Figure 5.7: The current distribution on the director N=3

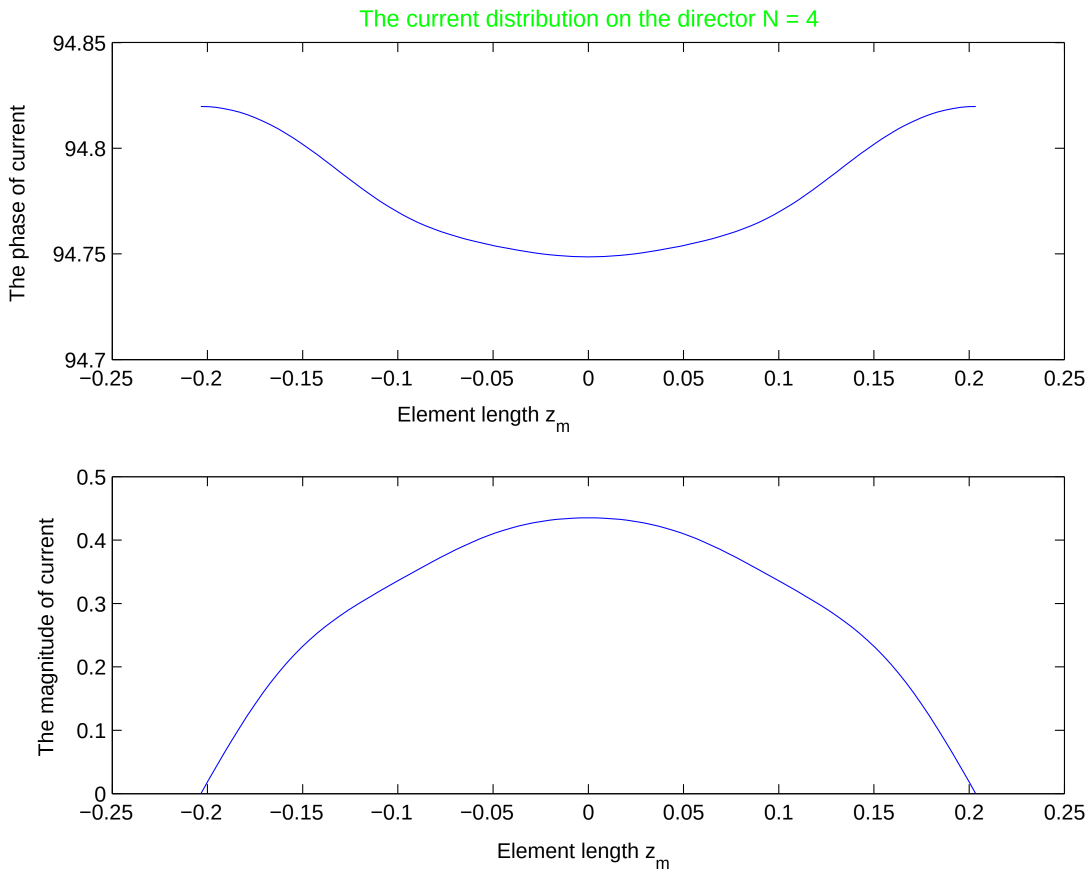

Figure 5.8: The current distribution on the director N=4

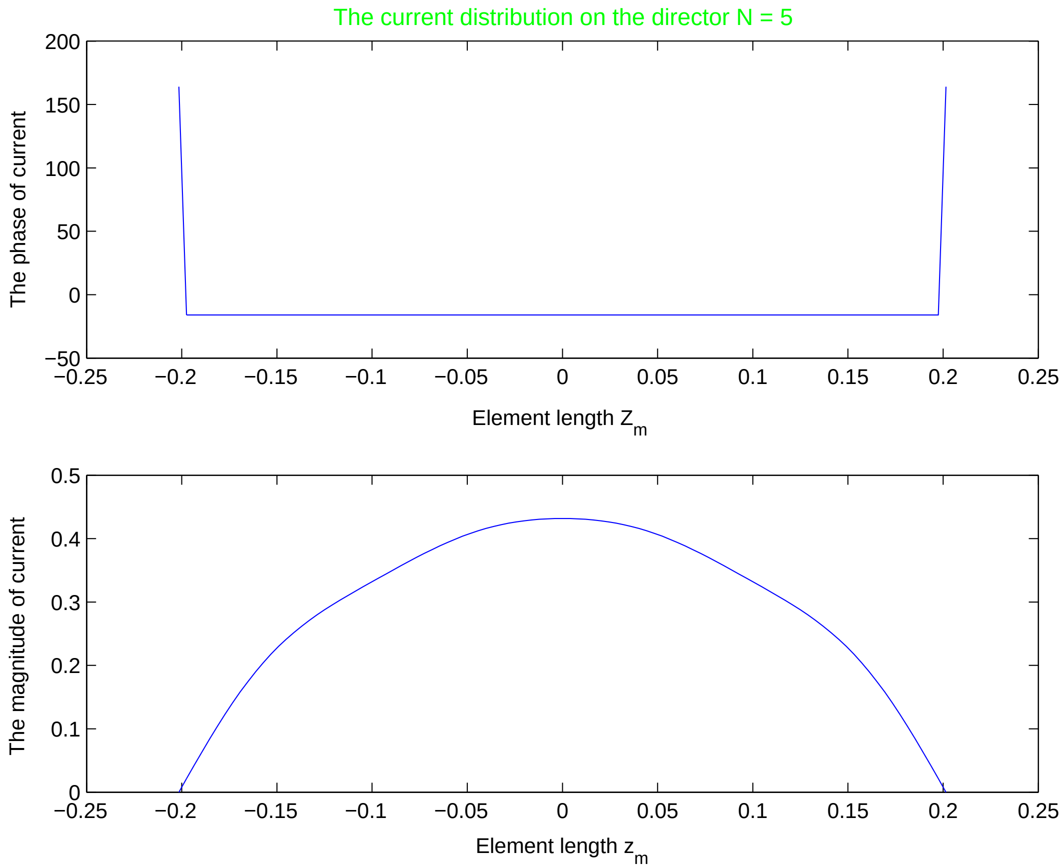

Figure 5.9: The current distribution on the director N=5

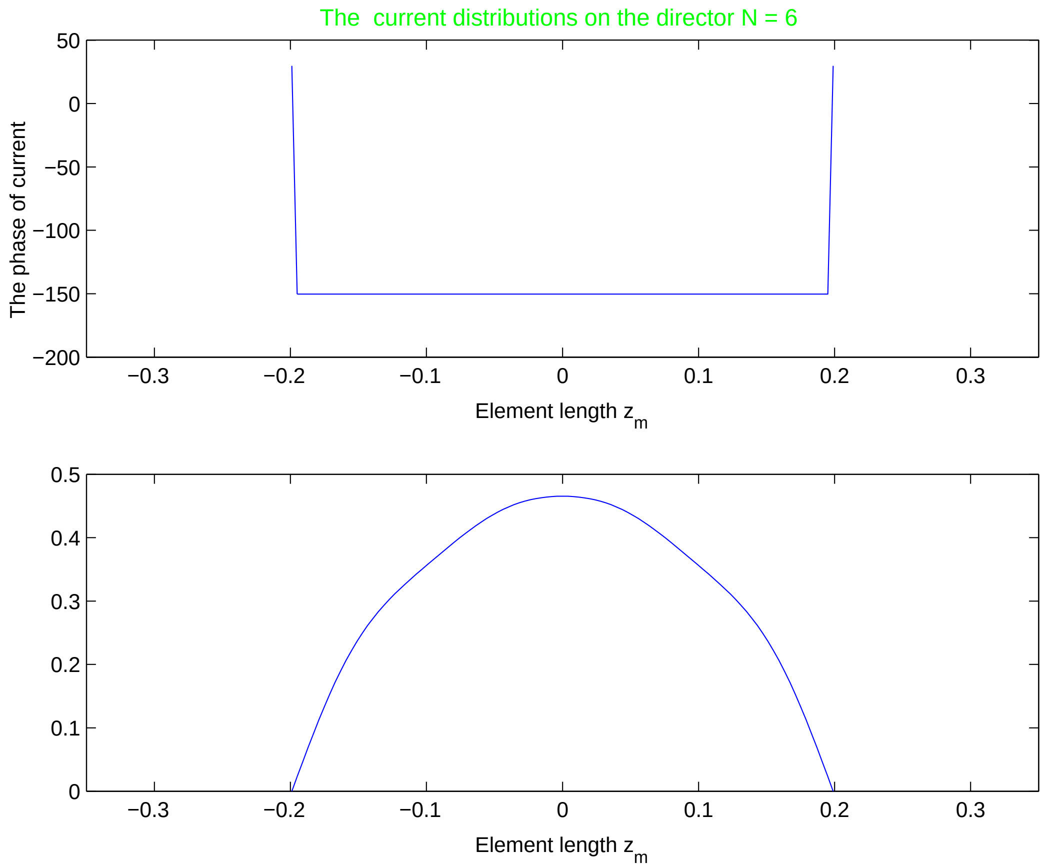

Figure 5.10: The current distribution on the director N=6

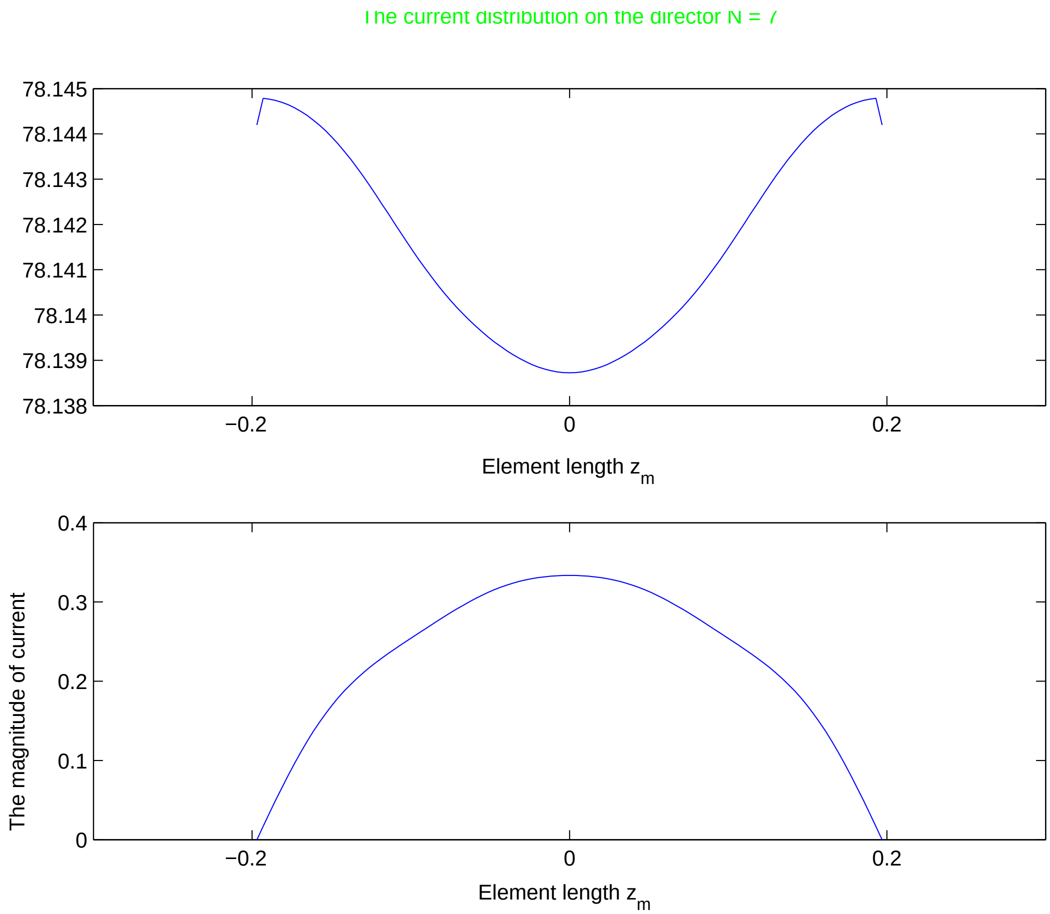

Figure 5.11: The current distribution on the director N=7

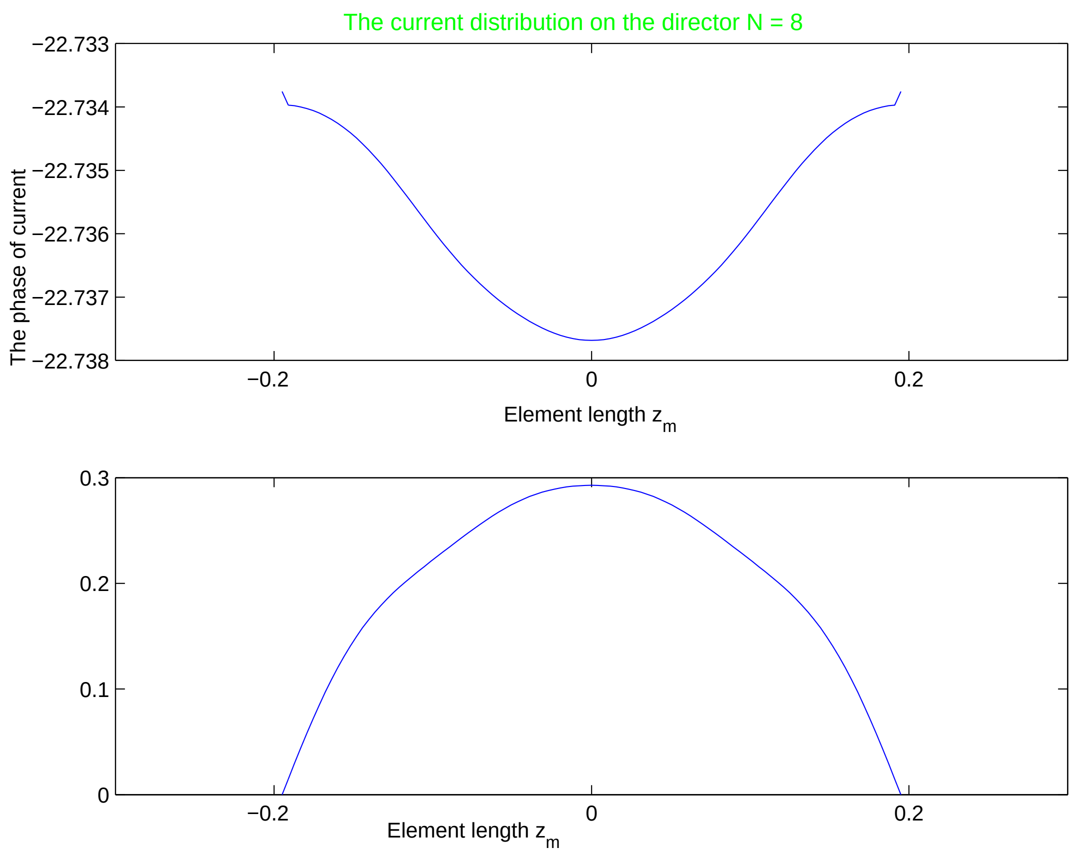

Figure 5.12: The current distribution on the director N=8

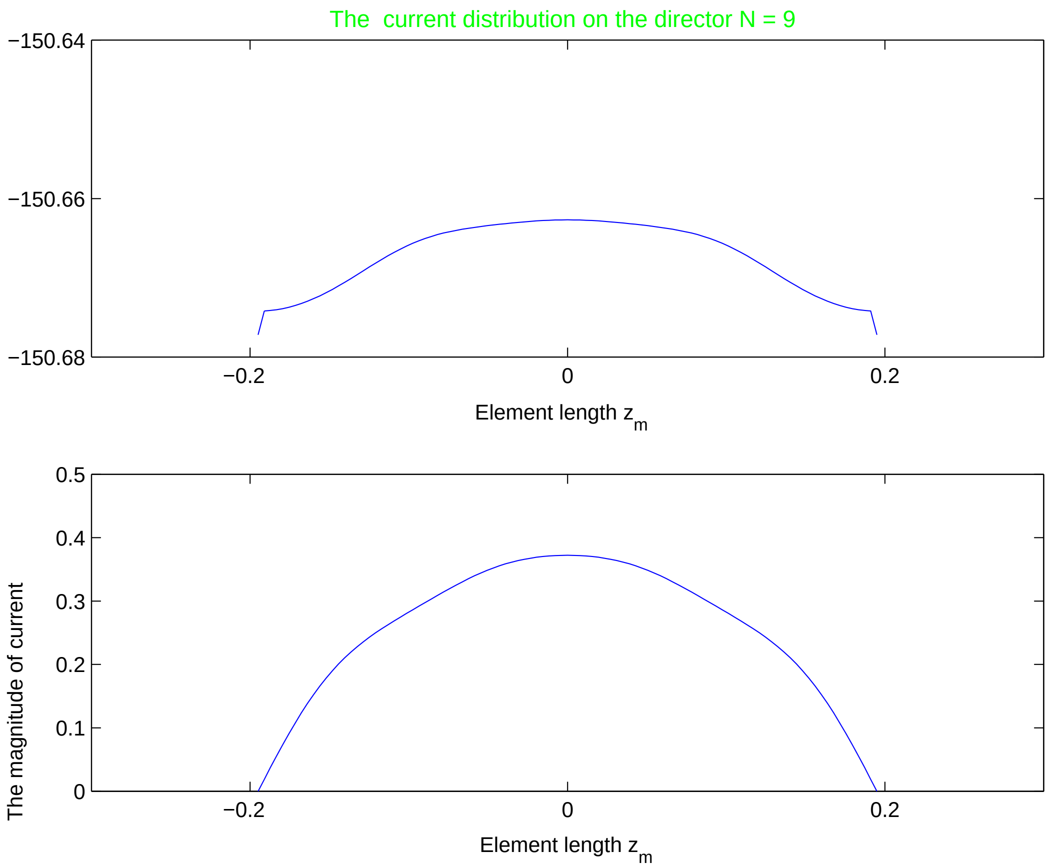

Figure 5.13: The current distribution on the director N=9

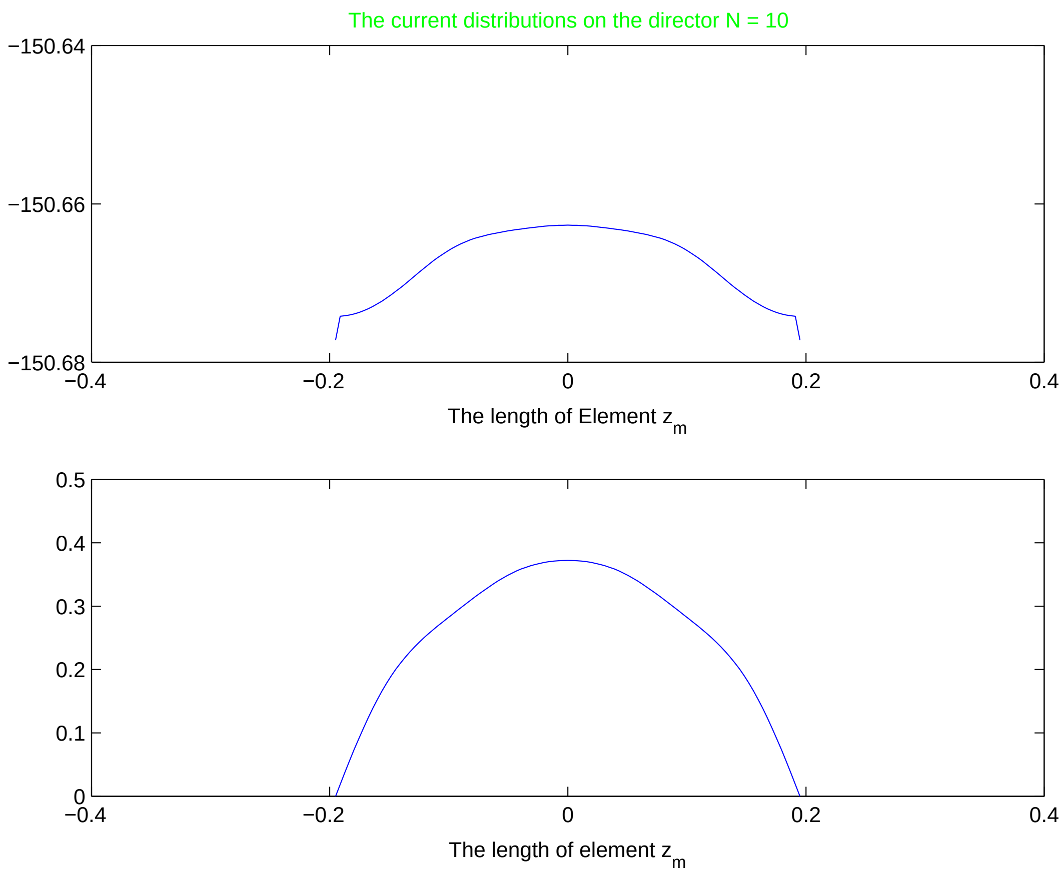

Figure 5.14: The current distribution on the director N=10

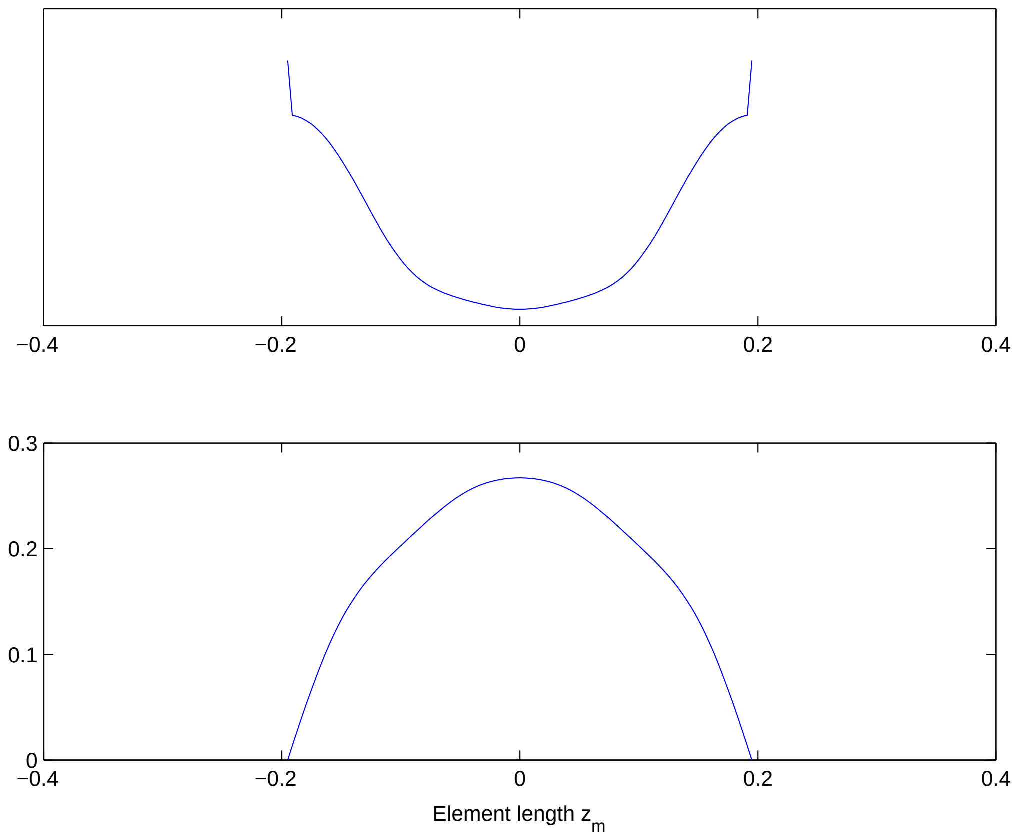

Figure 5.15: The current distribution on the director N=11

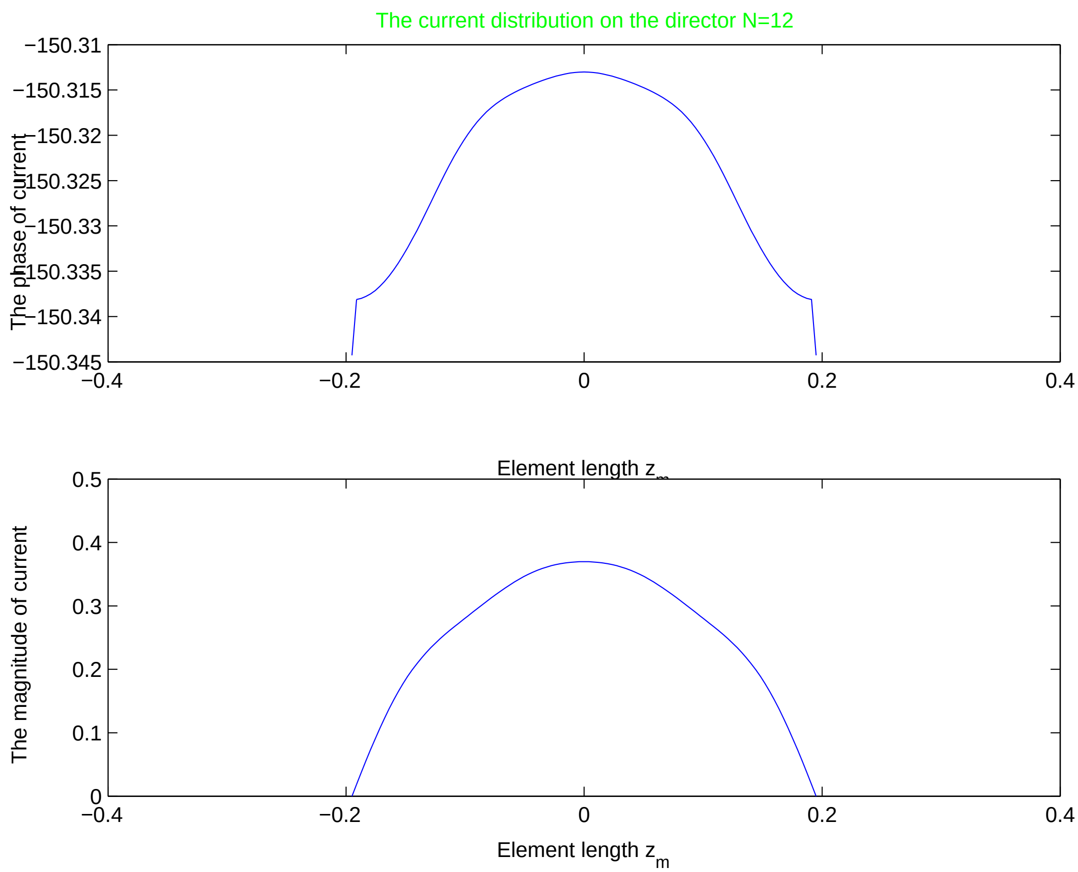

Figure 5.16: The current distribution on the director N=12

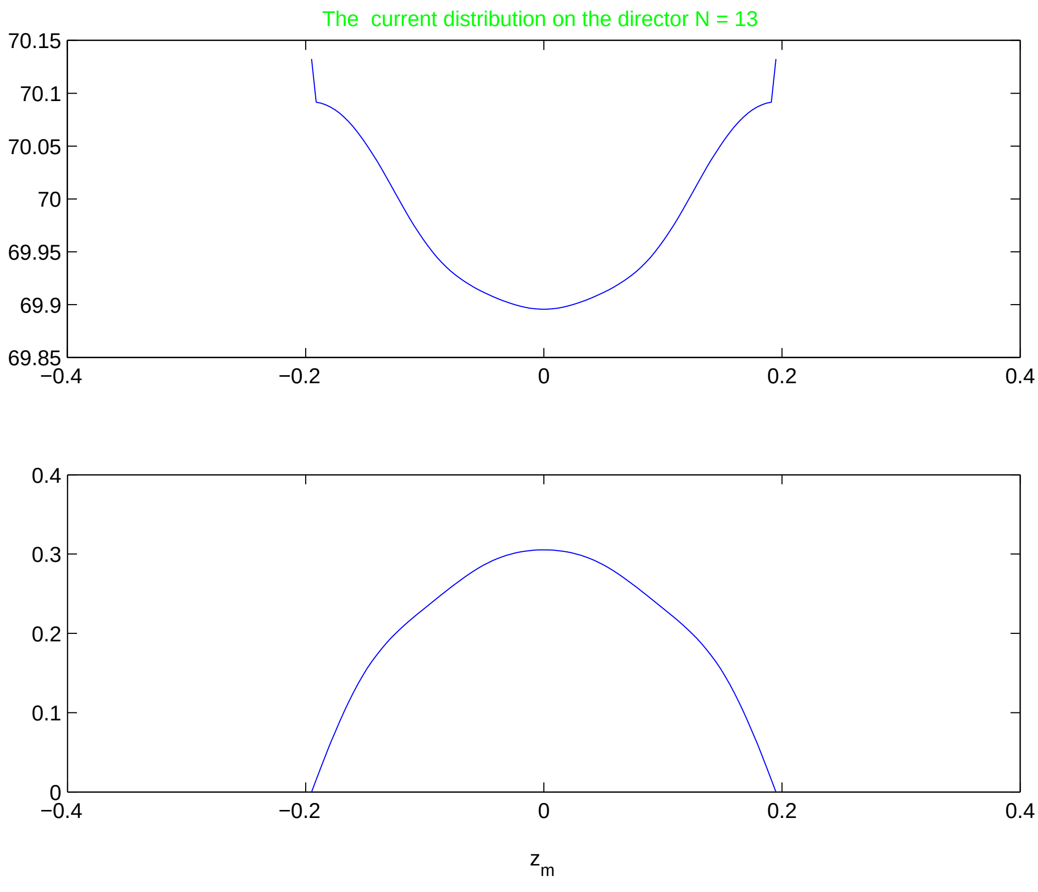

Figure 5.17: The current distribution on the director N=13

5.4 Characteristic variables

The characteristic variables of the designed NBS antenna can be calculated and listed in the following table.

Directivity	14.2106
Front-to-back ratio in e-plane	12.0779
Front-to-back ratio in h-plane	12.0811
3-db beamwidth in the e-plane	28.8770
3-db beamwidth in the h-plane	30.5268
Imput impedance	27.46051ohms

Table 5.1: The characteristics of this antenna design

Chapter 6

Conclusion

From National Bereau of Standards, we know 15-element yagi antenna has maximum directivity(gain) = $14.2db$. In our simulation result we obtain the directivity= 14.2106 is almost the same as NBS design. What's more, we got the resonable E(H)-field plot,complex current distributions, front-to-back ratio, 3db-beamwidth and input-impedance.

Appendix A

List of Routine

A.1 yagi.m

A.2 func.m

A.3 func2.m

Bibliography

[1] Balanis and Contantine A. *Advanced Engineering electromagnetics*. John Wiley– Sons, 1989.

[2] G.A.Thiele. Yagi-uda type antennas. *IEEE Trans. Antennas Propagat*, 17:21–31, 1969.

[3] Harrington. *Field computation by Moment Methods*. MacMillan, 1968.

[4] H.Schwetlick. Numerische losung nichtlinearer gleichungen. In *Deutscher Verlag der Wissenschaften*, 1979.

[5] N.K.Takla and L.C.Shen. Bandwidth of a yagi array with optimum directivity. *IEEE Trans. Antennas Propagat*, 25:913–914, 1977.

[6] P.P.Viezbicke. Yagi antenna design. *NBS Technical Note 688.*, 1976.

[7] Gordon W.J. and C.A. Hall. Transfinite element methods: Blending function interpolation over arbitary curved element domains. *Numer. Math.*, 21:109–129, 1973.