ASE 211 Homework 13

The motion of an object (a baseball for example), subjected to the forces of gravity and air resistance proportional to velocity, can be described by the system of ODEs:

$$x''(t) = -cv(t)x'(t)$$
$$y''(t) = -cv(t)y'(t) - g$$

where the speed of the object is $v = \sqrt{(x')^2 + (y')^2}$, $g = 32ft/s^2$ and $c = 0.002$ for a baseball. Use the 4th order Runge-Kutta method to solve this system and answer the question: does the ball clear a fence which is 400 ft from home plate and 10 ft tall if the initial conditions are $x(0) = y(0) = 0$, $x'(0) = y'(0) = 100ft/s$? What if $x'(0) = 160$ and $y'(0) = 83ft/s$?

Plot x and y versus t and x vs. y.

Hand in all m-files.

```
function [t,x,y]=assign13(h,z)
%
j=1;
t(j)=0;
x(j)=z(1);
y(j)=z(3);
while(x(j) <= 400)
    z=comprk4(h,t(j),z);
    j=j+1;
    t(j)=t(j-1)+h;
    x(j)=z(1);
    y(j)=z(3);
end

function y=comprk4(h,x,y)
k1=h*f(x,y);
k2=h*f(x+.5*h,y+.5*k1);
k3=h*f(x+.5*h,y+.5*k2);
k4=h*f(x+h,y+k3);
y=y+(k1+2*k2+2*k3+k4)/6;
```
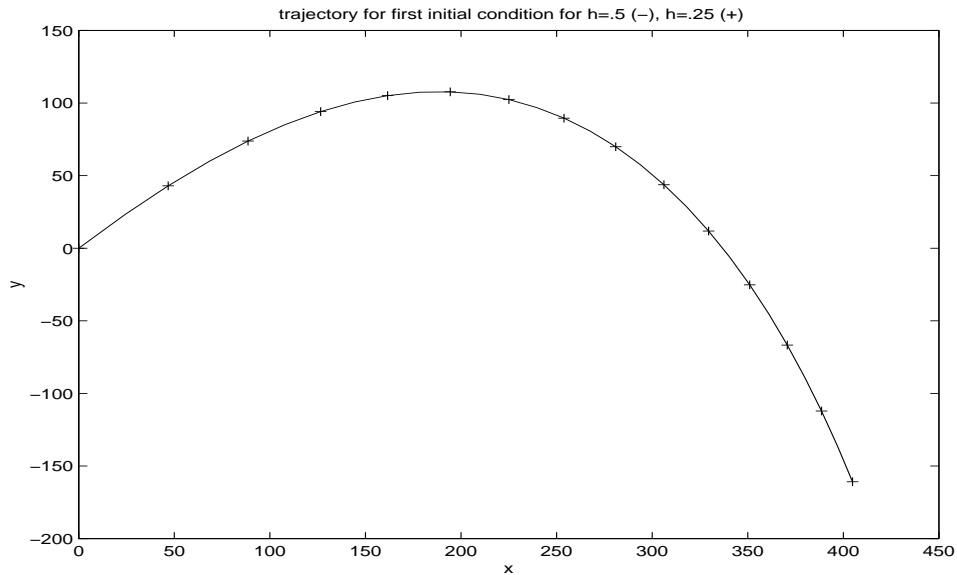
Figure 1: Comparison of trajectories for $h = .5$ and $h = .25$ for first initial condition

```
function r=f(x,y)
r(1)=y(2);
r(2)=-.002*sqrt(y(2)^2+y(4)^2)*y(2);
r(3)=y(4);
r(4)=-.002*sqrt(y(2)^2+y(4)^2)*y(4)-32;
```

For the first initial condition, we ran two cases, one with $h = .5$ and another with $h = .25$. The solutions are very similar, so we assume $h = .25$ is small enough to get an accurate solution. For the first initial condition, the baseball does not go over the fence, for the second initial condition, it does.
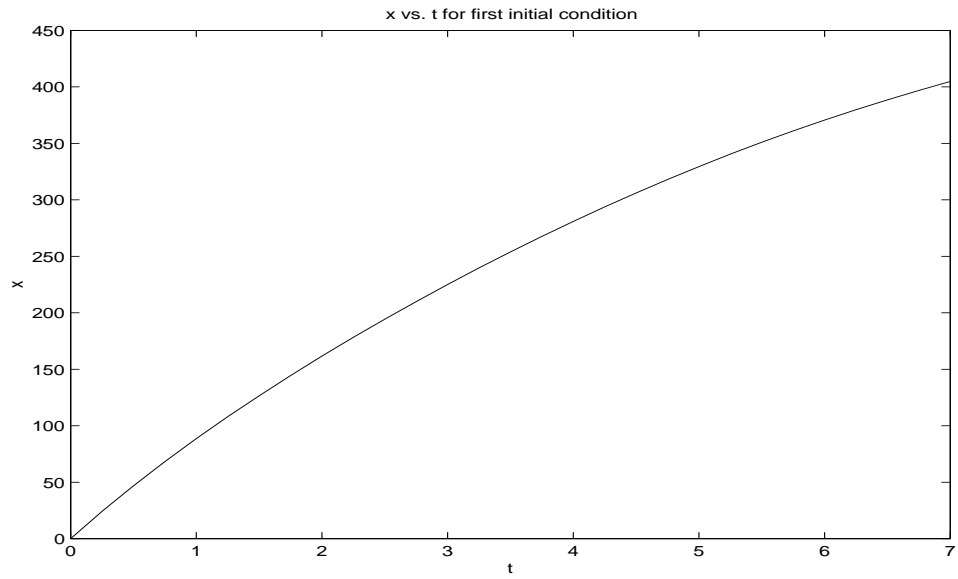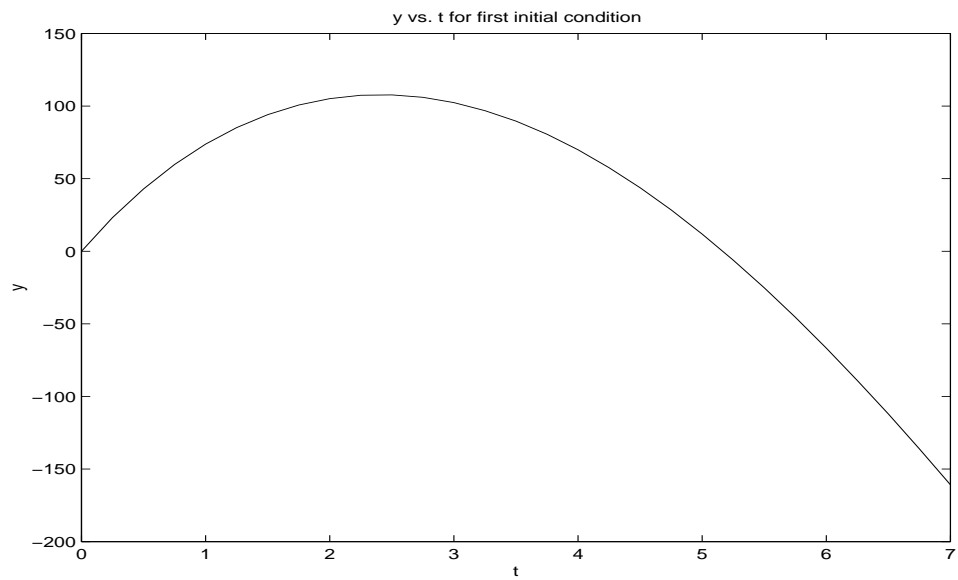
Figure 2: x vs. t for the first initial condition

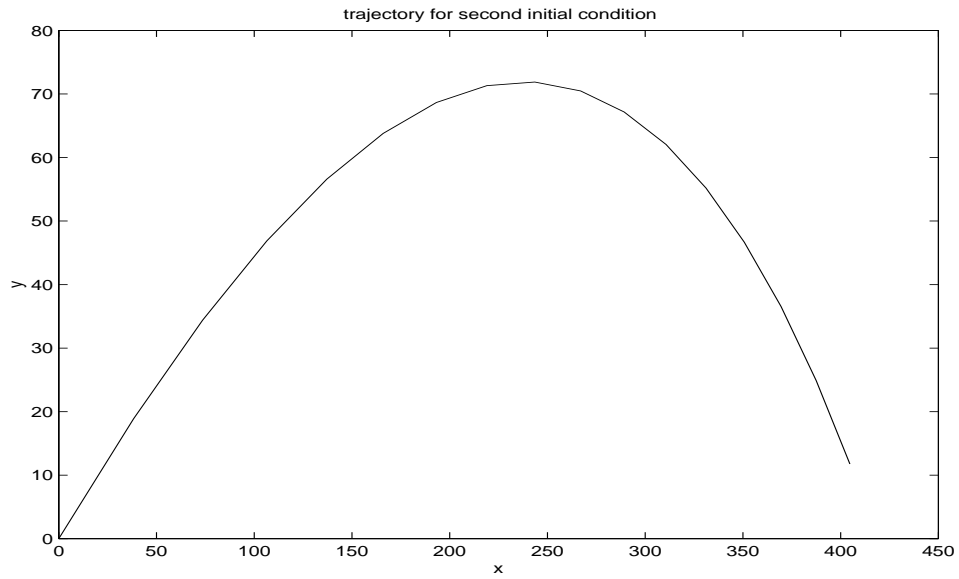Figure 3: y vs. t for the first initial condition

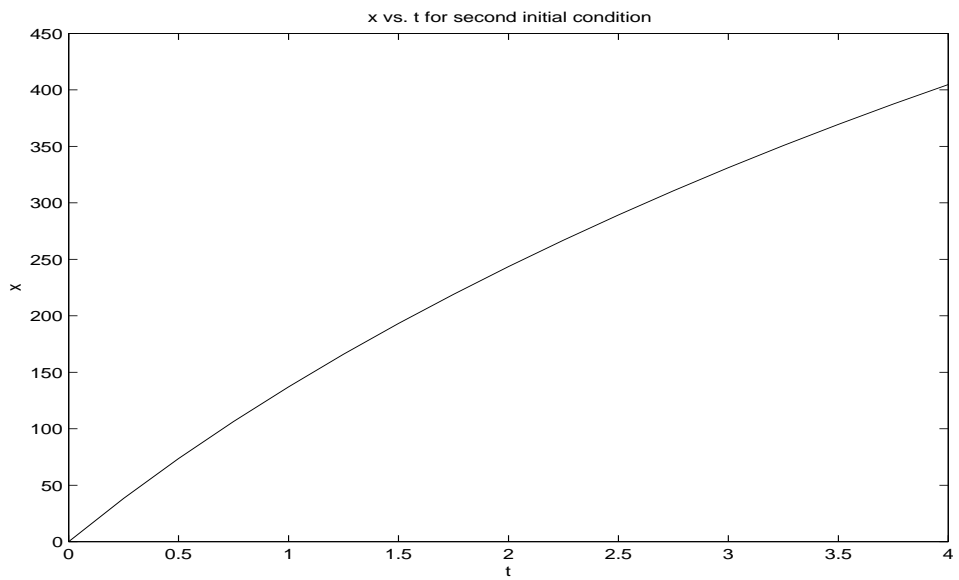Figure 4: Trajectory for the second initial condition
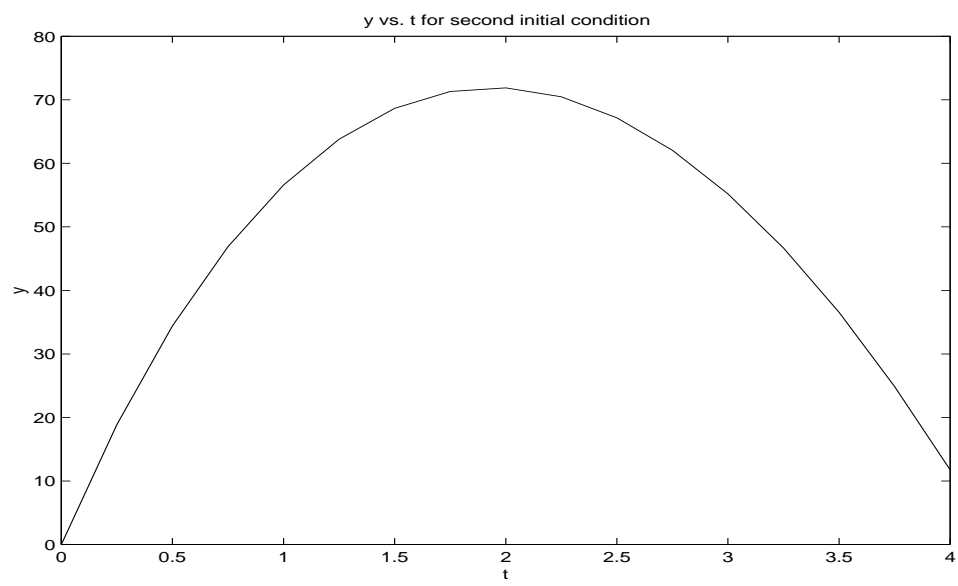

Figure 5: x vs. t for the second initial condition

Figure 6: y vs. t for the second initial condition