ASE 211 Homework 12 Solution

1. Write matlab m-files which implement the composite trapezoidal, Simpson's and midpoint rules. Verify that the composite trapezoidal and midpoint rules have error which goes to zero like $h^2$ by approximating the integral

$$\int_0^1 [3x^4 - x^3 + 12x^2]dx$$

and comparing the errors for $N = 10$, 20 and 40 (as discussed in class).

```
function sum=trap(a,b,N)
sum=f(a)+f(b);
h=(b-a)/N;
x=a;
for j=2:N
  x=x+h;
  sum=sum+2*f(x);
end
sum=sum*h/2;
```

```
function y=simp(a,b,N)
h=(b-a)/N;
y=f(a)+f(b);
x=a-h;
for i=2:2:N
   x=x+2*h;
   y=y+4*f(x);
end
x=a;
for i=3:2:N-1
   x=x+2*h;
   y=y+2*f(x);
end
y=y*h/3;
```

```
function sum=midpt(a,b,N)
sum=0;
```

```
h=(b-a)/N;
x=a-.5*h;
for j=1:N
  x=x+h;
  sum=sum+h*f(x);
end
```

The true integral has value 4.35.

Error for trapezoidal rule:

| N | error | ratio |
|---|---|---|
| 10 | .0275 | – |
| 20 | .0069 | 3.99 |
| 40 | .0017 | 4.06 |

Error for midpoint rule:

| N | error | ratio |
|---|---|---|
| 10 | .0137 | – |
| 20 | .0034 | 4.03 |
| 40 | .00086 | 3.95 |

In both cases, the error goes down by a factor of 2 as N is doubled, indicating $h^2$ convergence.

2. Apply your composite trapezoidal and Simpson's codes to problem A11.1 in the book.

```
function sum=trapdata(x,f,N)
%
% uses trapezoidal rule to integrate with given data (x(i),f(i))
%
sum=f(1)+f(N+1);
h=(x(N+1)-x(1))/N;
for j=2:N
  sum=sum+2*x(j)*f(j);
end
sum=sum*pi*h;
```

```
function y=simpdata(x,f,N)
%
% uses Simpson's rule to integrate with given data (x(i),f(i))
```

```
% Note:  N must be even
%
h=(x(N+1)-x(1))/N;
y=f(1)+f(N+1);
for i=2:2:N
   y=y+4*f(i);
end
for i=3:2:N-1
   y=y+2*f(i);
end
y=y*h/3;
```

Using the data in problem A11.1, the trapezoidal rule obtains the answer 1.4929, while Simpson's rule gets 1.4879. Using the function $V(r) = 1 - r^2$

$$2\pi \int_0^1 r(1 - r^2)dr = 1.5708.$$

The trapezoidal rule obtains 1.5551 and Simpson's rule obtains 1.5708. Simpson's rule is exact for this integral.