

ASE 211 Homework 7

Due: 12:00 noon, Friday, October 20.

Given the following data

i	$x_i$	$y_i$
1	0	4
2	1.1	-1.5
3	1.7	0.5
4	2.2	1.5

1. Compute the cubic spline matrix  $A$  and the right hand side  $\mathbf{g}$ , and solve for the vector of second derivatives  $\mathbf{s}$ .

For the data above,  $h_1 = 1.1$ ,  $h_2 = .6$  and  $h_3 = .5$ , and  $n = 4$ , so the spline matrix is the 2x2 matrix

$$\begin{bmatrix} 2(h_1 + h_2) & h_2 \\ h_2 & 2(h_2 + h_3) \end{bmatrix} = \begin{bmatrix} 3.4 & .6 \\ .6 & 2.2 \end{bmatrix}$$

The right hand side is

$$6 \begin{bmatrix} \frac{y_3 - y_2}{h_2} - \frac{y_2 - y_1}{h_1} \\ \frac{y_4 - y_3}{h_3} - \frac{y_3 - y_2}{h_2} \end{bmatrix} = \begin{bmatrix} 50 \\ -8 \end{bmatrix}$$

Solving the system we get  $S_2 = 16.1236$ ,  $S_3 = -8.0337$ . Also remember that  $S_1 = S_4 = 0$ .

2. Use  $\mathbf{s}$  to compute the coefficients of the cubic spline.

$$\begin{aligned} a_1 &= \frac{S_2 - S_1}{6h_1} \\ &= 2.4430 \\ b_1 &= S_1/2 \\ &= 0 \\ c_1 &= \frac{y_2 - y_1}{h_1} - \frac{S_2 - S_1}{6}h_1 - \frac{S_1}{2}h_1 \\ &= -7.9560 \\ d_1 &= y_1 \\ &= 4 \end{aligned}$$

$$a_2 = -6.7104$$

$$b_2 = 8.0618$$

$$c_2 = .9120$$

$$d_2 = -1.5$$

$$a_3 = 2.6779$$

$$b_3 = -4.0169$$

$$c_3 = 3.3390$$

$$d_3 = .5$$

3. Use matlab to plot the cubic spline between  $x_1$  and  $x_n$  using the following *m - file*:

```
function plot_spline(a,b,c,d,n,x)
% function which plots a spline given its coefficients
% a(i),b(i),c(i),d(i), i=1 to n-1
% and the data points x(i), i=1 to n
%
aa=x(1);
bb=x(n);
h=(bb-aa)/100;
for i=1:100
    xx(i)=aa+i*h;
    for j=1:n-1
        if ((x(j) <= xx(i)) & (xx(i) <= x(j+1)))
            yy(i)=a(j)*(xx(i)-x(j))^3+b(j)*(xx(i)-x(j))^2+c(j)*(xx(i)-x(j))+d(j);
        end
    end
end
end
plot(xx,yy)
```

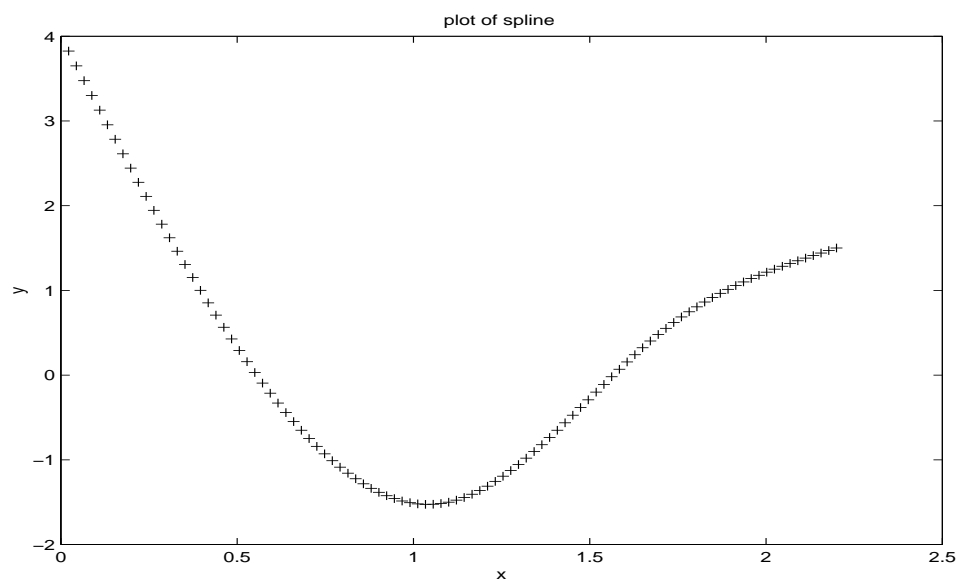


Figure 1: Plot of spline