ASE 211 Homework 4

Due: 12:00 noon, Friday, September 29. Put assignments in the drawer on the third floor of WRW marked 'ASE 211.'

1. By hand, compute the LU decomposition of the following matrix:

$$A = \begin{bmatrix} 3 & -1 & 4 \\ -6 & 4 & -1 \\ 18 & -2 & 39 \end{bmatrix}.$$

$$u_{11} = a_{11} = 3, \quad u_{12} = a_{12} = -1, \quad u_{13} = a_{13} = 4$$
$$l_{21} = a_{21}/u_{11} = -2, \quad l_{31} = a_{31}/u_{11} = 6$$
$$u_{22} = a_{22} - l_{21}u_{12} = 2, \quad u_{23} = a_{23} - l_{21}u_{13} = 7$$
$$l_{32} = (a_{32} - l_{31}u_{12})/u_{22} = 2$$
$$u_{33} = a_{33} - l_{31}u_{13} - l_{32}u_{23} = 1$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 6 & 2 & 1 \end{bmatrix}.$$

$$U = \begin{bmatrix} 3 & -1 & 4 \\ 0 & 2 & 7 \\ 0 & 0 & 1 \end{bmatrix}.$$

2. For the matrix in problem 1, use forward and backward substitution to solve $A\mathbf{x} = \mathbf{b}$, where

$$\mathbf{b} = \begin{bmatrix} 20 \\ -26 \\ 150 \end{bmatrix}.$$

Forward substitution: $L\mathbf{y} = \mathbf{b}$

$$y_1 = b_1 = 20$$
$$y_2 = b_2 - l_{21}y_1 = 14$$
$$y_3 = b_3 - l_{31}y_1 - l_{32}y_2 = 2$$

Backward substitution: $U\mathbf{x} = \mathbf{y}$

$$x_3 = y_3/u_{33} = 2$$
$$x_2 = (y_2 - u_{23}x_3)/u_{22} = 0$$
$$x_1 = (y_1 - u_{13}x_3 - u_{12}x_2)/u_{11} = 4$$

3. Create a matlab m-file called ludecomp.m (as discussed in class), which takes as input a matrix A, and returns the LU decomposition of A. Test your m-file on the matrix in problem 1.

```
function [L,U]=ludecomp(A)
%  creates the LU decomposition of matrix A
%
% find the size of A
n=size(A,1);
for i=1:n
   L(i,i)=1;
end
for j=1:n
  for i=j:n
    U(j,i)=A(j,i);
    for k=1:j-1
       U(j,i)=U(j,i)-L(j,k)*U(k,i);
    end
  end
  for i=j+1:n
    L(i,j)=A(i,j);
    for k=1:j-1
       L(i,j)=L(i,j)-L(i,k)*U(k,j);
    end
    L(i,j)=L(i,j)/U(j,j);
  end
end

>> A=[3 -1 4;-6 4 -1; 18 -2 39]

A =

      3     -1      4
```

```
    -6      4     -1
    18     -2     39

>> [L,U]=ludecomp(A)

L =

     1      0      0
    -2      1      0
     6      2      1


U =

     3     -1      4
     0      2      7
     0      0      1

>> diary
```

4. Write Matlab *m*-files forsolve.m and backsolve.m which perform forward and backward substitution, given the LU decomposition of the matrix. Test your *m*-files on the system given in problems 1 and 2.

```
function x=forsolve(A,b)
% solves for Ax=b, where A is lower triangular
%
n=size(A,1);
for i=1:n
  x(i)=b(i);
  for k=1:i-1
     x(i)=x(i)-A(i,k)*x(k);
  end
  x(i)=x(i)/A(i,i);
end

function x=backsolve(A,b)
% solves for Ax=b, where A is upper triangular
%
```

```
n=size(A,1);
for i=n:-1:1
  x(i)=b(i);
  for k=n:-1:i+1
     x(i)=x(i)-A(i,k)*x(k);
  end
  x(i)=x(i)/A(i,i);
end
```

```
>> L

L =

     1     0     0
    -2     1     0
     6     2     1

>> U

U =

     3    -1     4
     0     2     7
     0     0     1

>> b=[20; -26; 150]

b =

    20
   -26
   150

>> y=forsolve(L,b)

y =

    20    14     2
```

```
>> x=backsolve(U,y)

x =

     4     0     2

>> diary
```

5. Use the Matlab code you have written in problems 3 and 4 to solve the system in problem A3.4 in the book. First take $W_3 = 100$, then $W_3 = 50$, 150 and 200. Be sure to output the solution for each value of W_3. Remember you only have to do the LU decomposition once.

```
>> d=sqrt(2)/2

d =

    0.7071

>> clear A
>> A=[1 0 0 0 d 0;0 1 0 1 d 0;0 0 1 0 0 d;0 0 0 -1 0 -d;0 0 0 0 -d -d;0 0 0 0 -d d]

A =

    1.0000         0         0         0    0.7071         0
         0    1.0000         0    1.0000    0.7071         0
         0         0    1.0000         0         0    0.7071
         0         0         0   -1.0000         0   -0.7071
         0         0         0         0   -0.7071   -0.7071
         0         0         0         0   -0.7071    0.7071

>> [L,U]=ludecomp(A)

L =

     1     0     0     0     0     0
     0     1     0     0     0     0
     0     0     1     0     0     0
     0     0     0     1     0     0
```

```
         0      0      0      0      1      0
         0      0      0      0      1      1


U =

   1.0000        0        0        0   0.7071        0
        0   1.0000        0   1.0000   0.7071        0
        0        0   1.0000        0        0   0.7071
        0        0        0  -1.0000        0  -0.7071
        0        0        0        0  -0.7071  -0.7071
        0        0        0        0        0   1.4142


>> w3=50;
>> b=[0;0;0;0;w3;0]

b =

     0
     0
     0
     0
    50
     0

>> y=forsolve(L,b);
>> x=backsolve(U,y)

x =

   25.0000    0.0000   25.0000   25.0000  -35.3553  -35.3553

>> w3=150;
>> b=[0;0;0;0;w3;0]

b =

     0
```

```
     0
     0
     0
   150
     0

>> y=forsolve(L,b);
>> x=backsolve(U,y)

x =

   75.0000         0   75.0000   75.0000 -106.0660 -106.0660

>> w3=200;
>> b=[0;0;0;0;w3;0]

b =

     0
     0
     0
     0
   200
     0

>> y=forsolve(L,b);
>> x=backsolve(U,y)

x =

  100.0000    0.0000  100.0000  100.0000 -141.4214 -141.4214

>> diary
```