# Isogeometric Analysis: Past, Present, Future


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Institute for Computational Engineering and Sciences (ICES)

The University of Texas at Austin


Collaborators:

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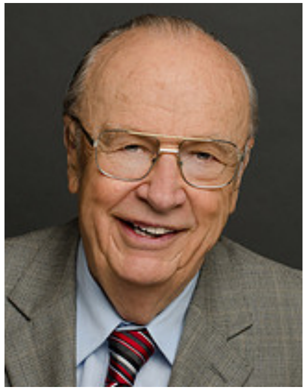
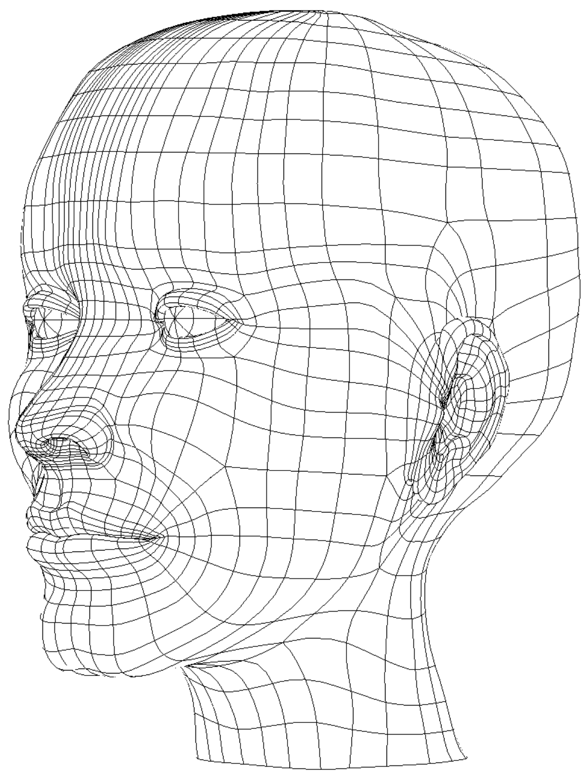


Babuška Forum

September 23rd, 2016

# Outline

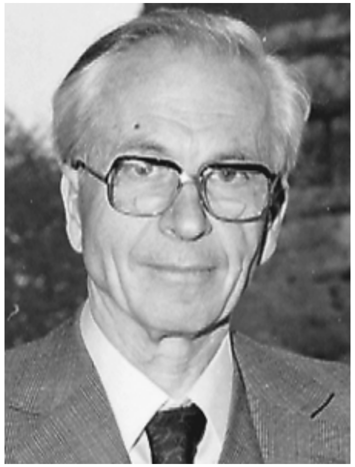
- FEA, since 1956
- IGA, since 2005
- B-splines, NURBS
- Collocation
- Quadrature
- Applications
  - Aortic valves
  - Boiling
  - Ductile fracture
- Summary and comments on new ideas



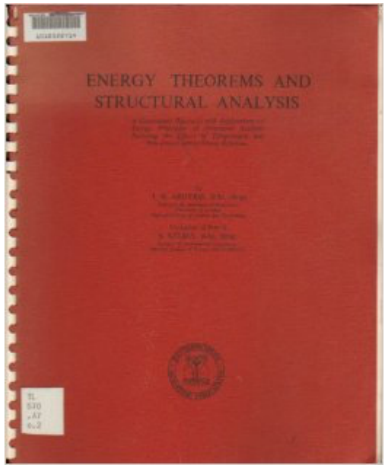
# The Finite Element Method Historical Publication Data

## The First 30 Years, 1956-1985

# Why 1956?



John Argyris, 1913 – 2004

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Ray Clough, 1920 –

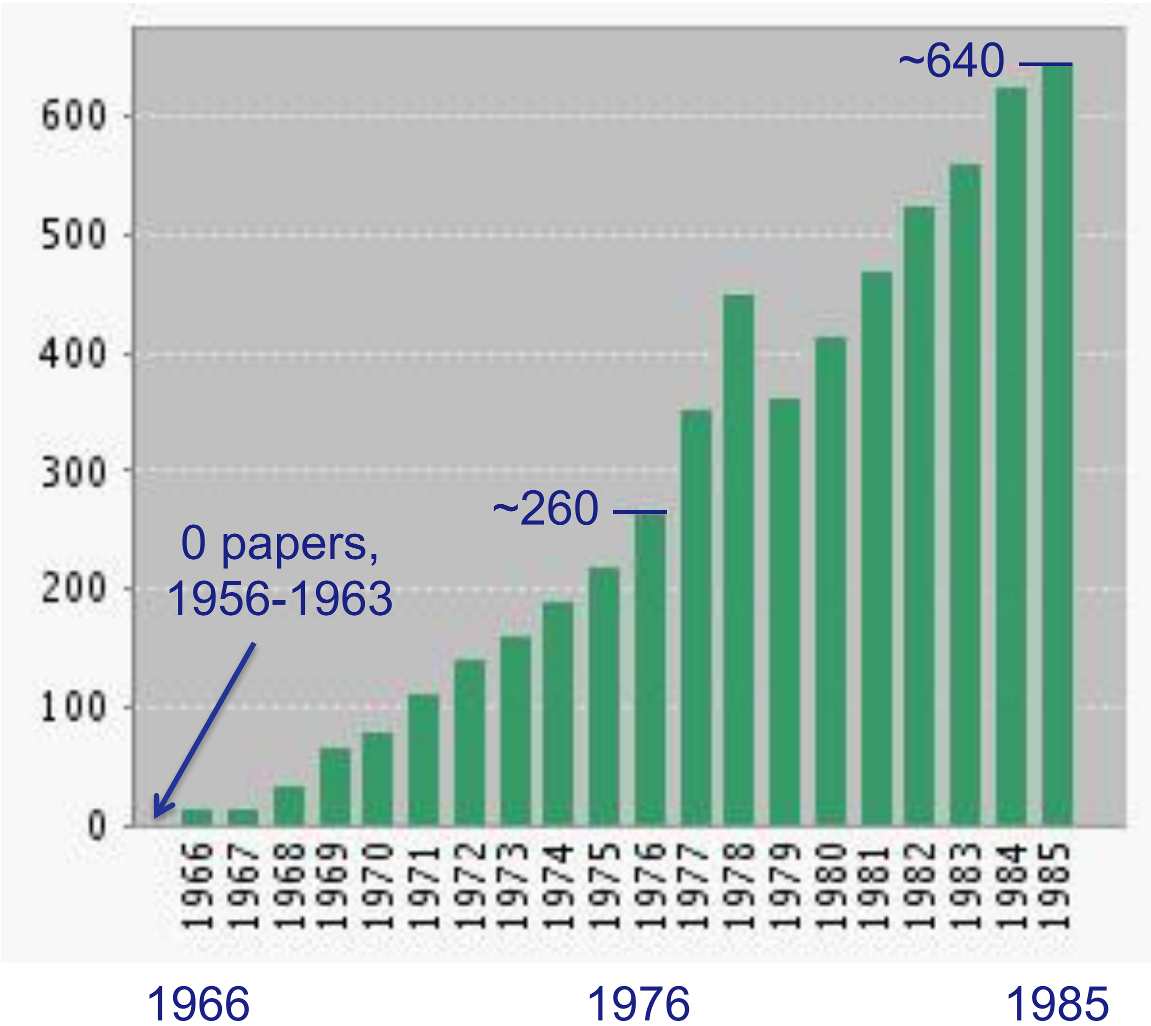
JOURNAL OF THE
AERONAUTICAL SCIENCES

VOLUME 23 SEPTEMBER, 1956 NUMBER 9

Stiffness and Deflection Analysis of Complex Structures

M. J. TURNER,* R. W. CLOUGH,† H. C. MARTIN,‡ AND L. J. TOPP**

# Number of FE Papers, 1956-1985

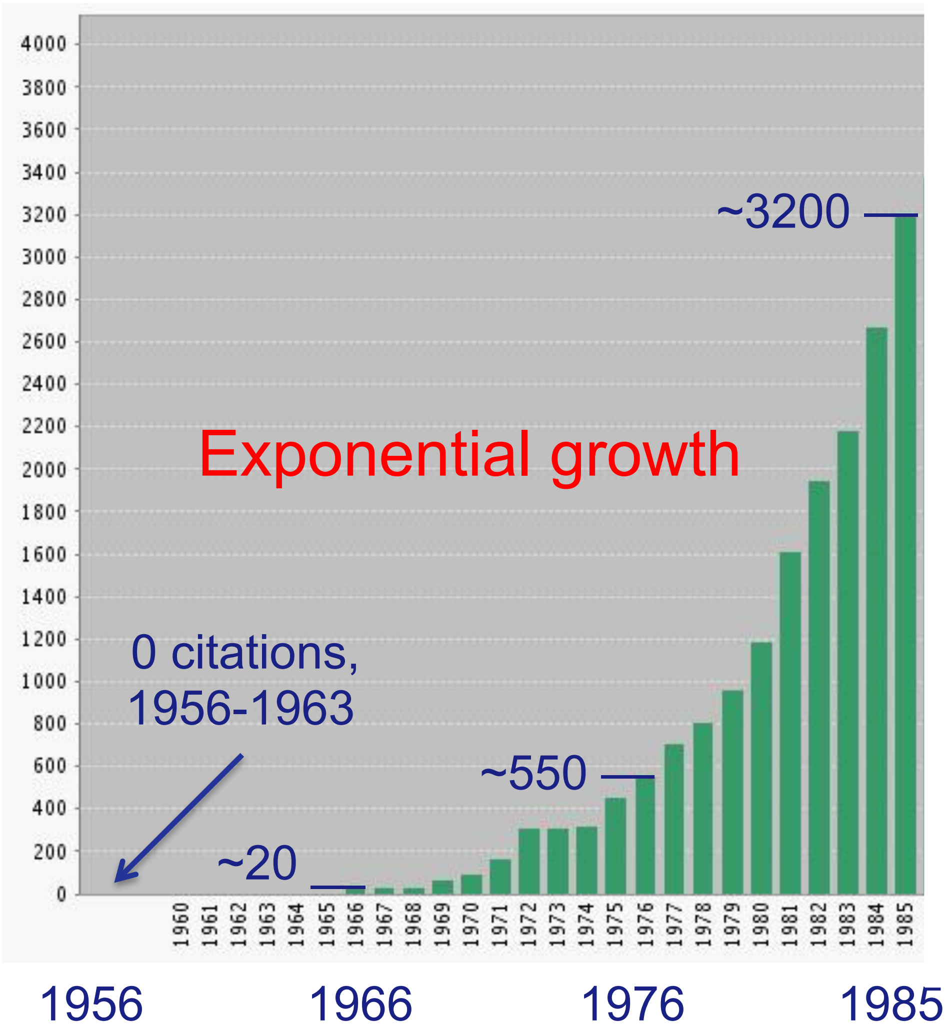



ISI Thomson-Reuters search

All data bases

Topic: Finite Element

# Number of FE Citations, 1956-1985



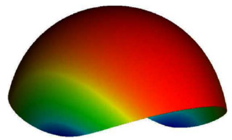

ISI Thomson-Reuters search

All data bases

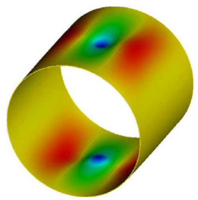
Topic: Finite Element

# Isogeometric Analysis
# Historical Publication Data

## The First 10 Years, 2006-2015

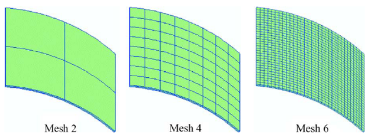


“Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement”



T.J.R. Hughes, J.A. Cottrell, Y. Bazilevs

*Computer Methods in Applied Mechanics and Engineering*




Volume 194, Pages 4135-4195 (Oct. 1, 2005)

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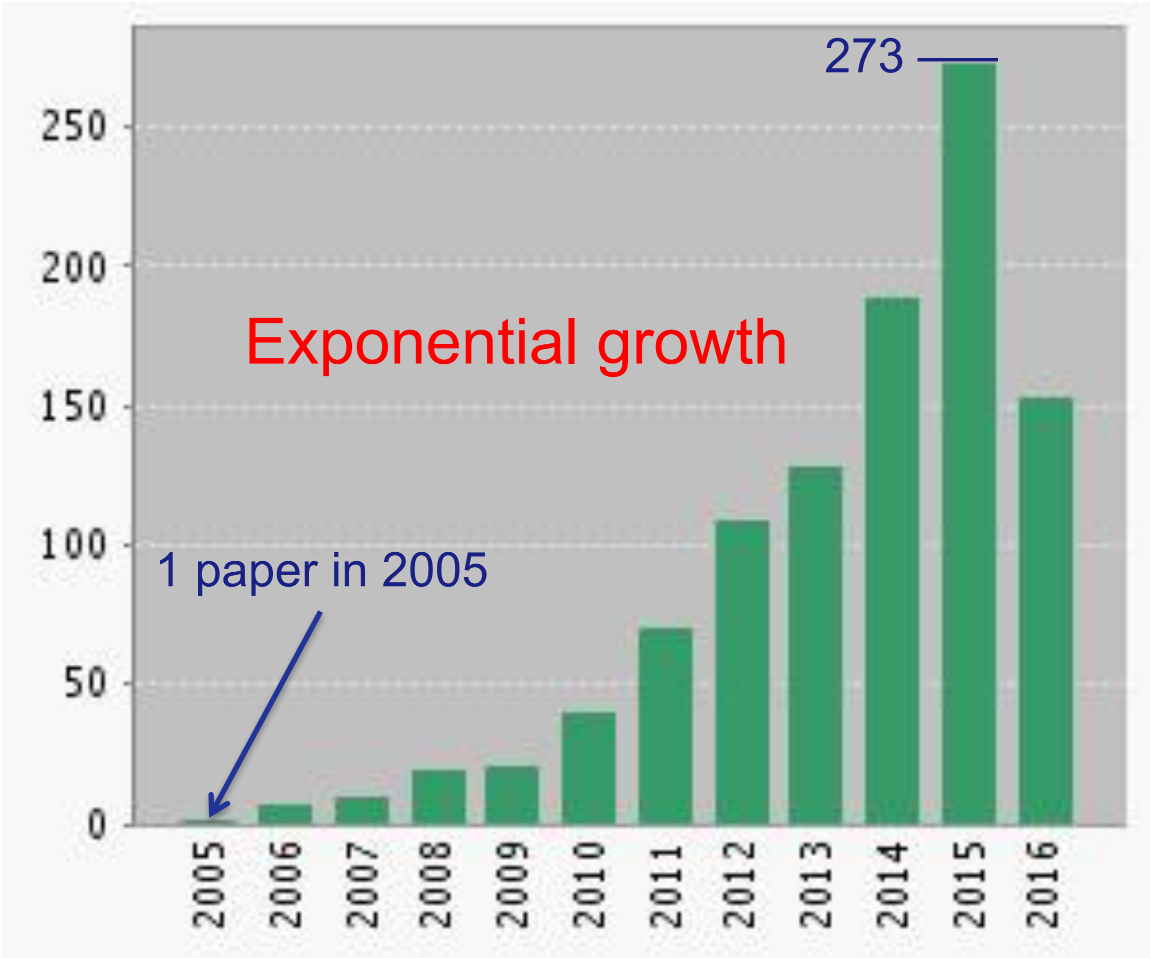
## Impact:

- *Still* the most downloaded *CMAME* paper
- Google Scholar: 2333 total, 451 last year (September 23, 2016)
- Thomson Reuters: 1128 total, 278 last year (September 23, 2016)

# Number of IGA Papers, 2006-2015




2006 2015

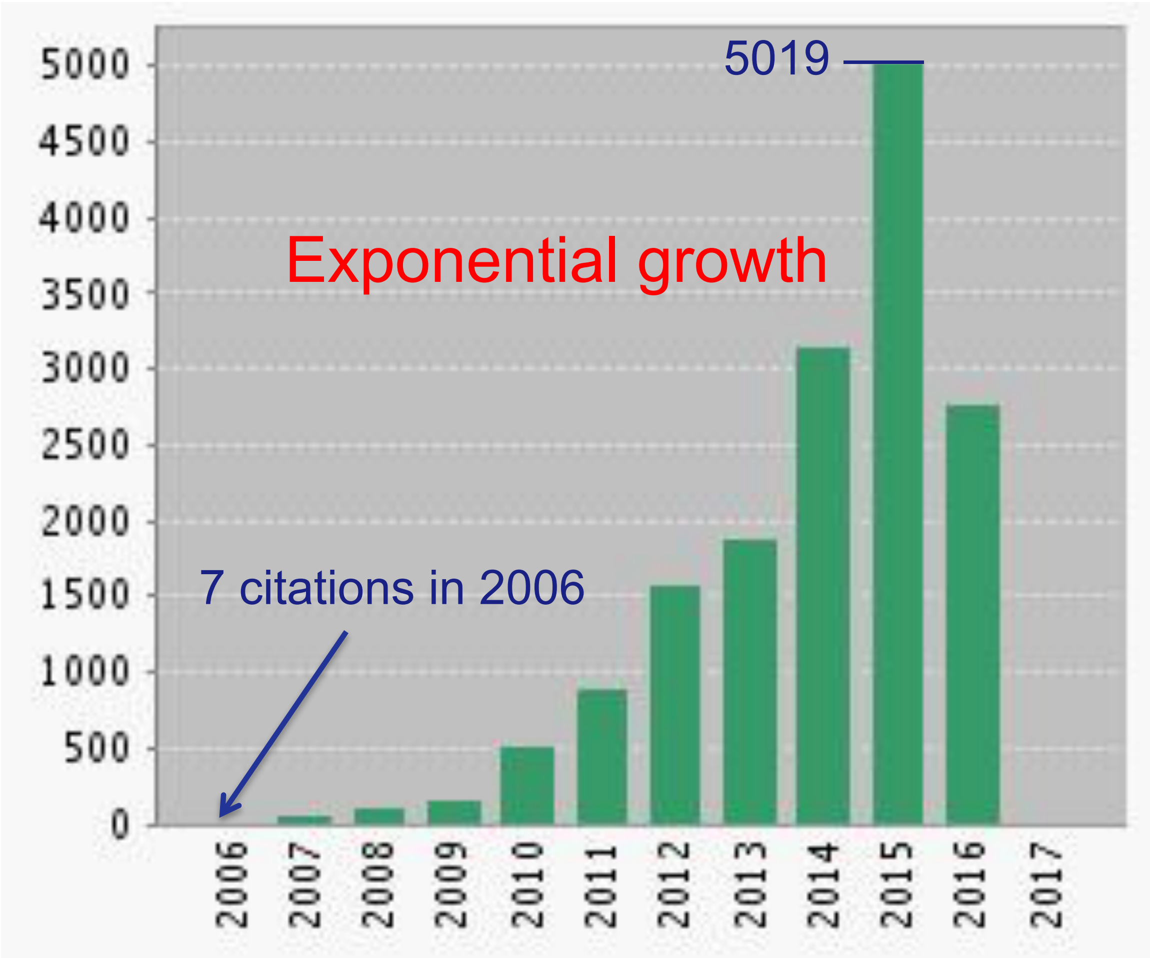
ISI Thomson-Reuters search

All data bases

Topic: Isogeometric Analysis

Date: September 23, 2016

# Number of IGA Citations, 2006-2015




2006 2015

ISI Thomson-Reuters search

All data bases

Topic: Isogeometric Analysis

Date: September 23, 2016

# Comparisons are odious*

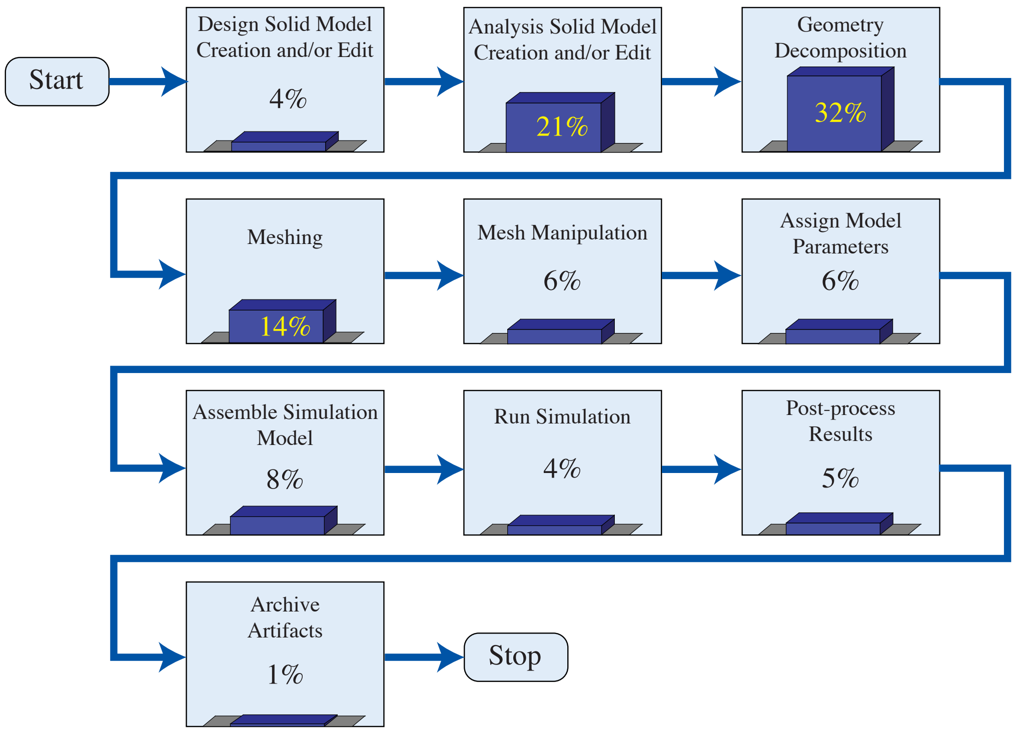
- Papers per year:
  - IGA 10th year (273) ≈ FEA 20th year (260)

- Citations per year:
  - IGA 10th year (5019) > FEA 30th year (3200)

*John Lydgate in his *Debate between the horse, goose, and sheep*, circa 1440

# Engineering *Analysis* Process

- Finite Element Analysis (FEA) models are created from CAD representations

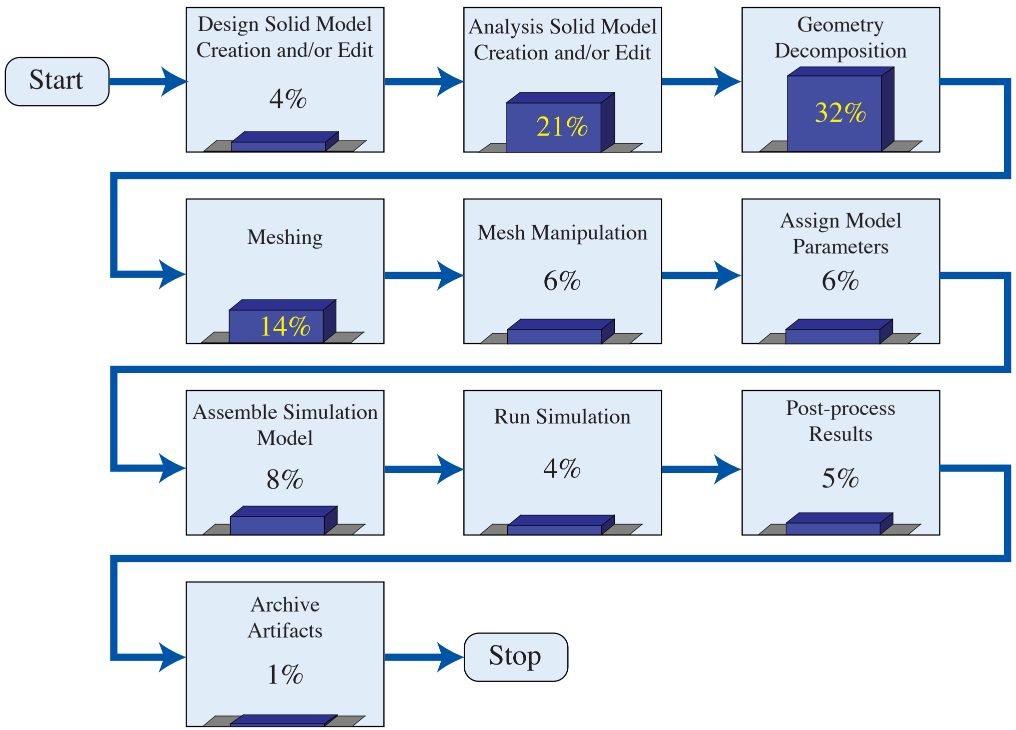



(Michael Hardwick and Robert Clay, Sandia National Laboratories)

# Engineering *Analysis* Process

- Finite Element Analysis (FEA) models are created from CAD representations

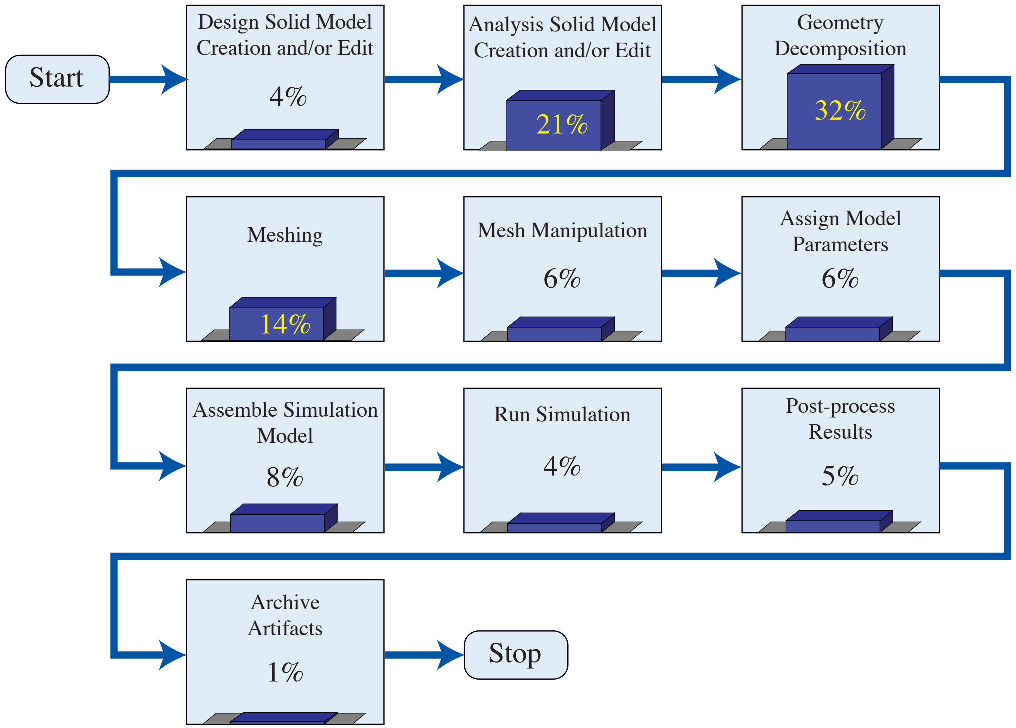
- Fixing CAD geometry and creating FEA models accounts for more than 80% of overall analysis time and is a major engineering *bottleneck*




(Michael Hardwick and Robert Clay, Sandia National Laboratories)

# Engineering *Analysis* Process

- Finite Element Analysis (FEA) models are created from CAD representations
- Fixing CAD geometry and creating FEA models accounts for more than 80% of overall analysis time and is a major engineering *bottleneck*
- The FEA mesh is also only an *approximate* geometry

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# Engineering *Analysis* Process

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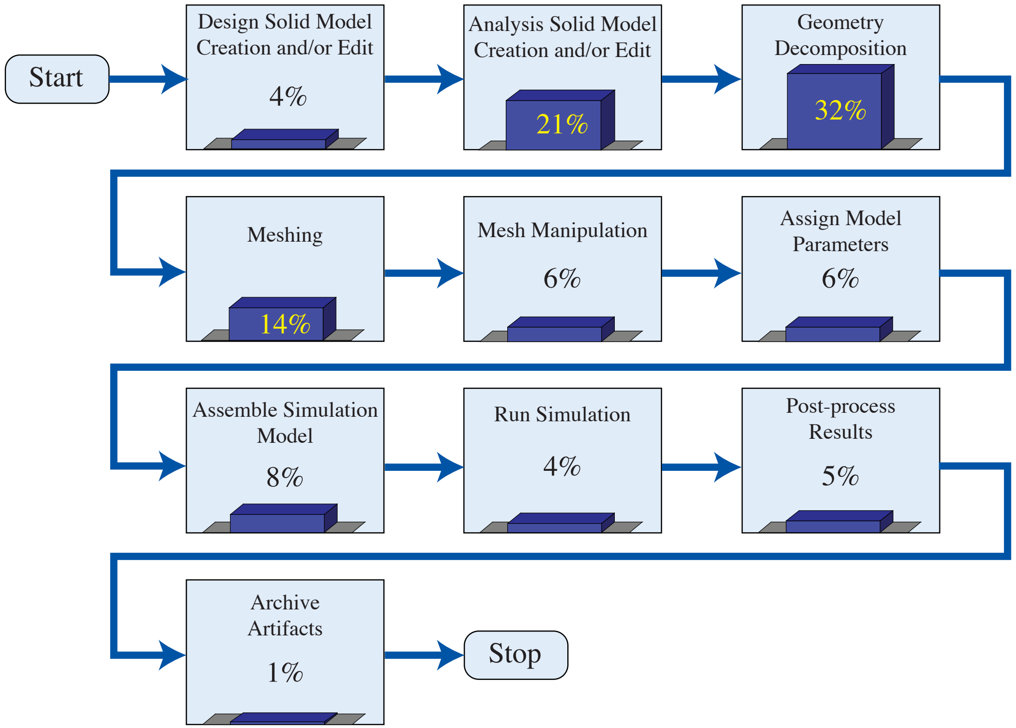



(Michael Hardwick and Robert Clay, Sandia National Laboratories)

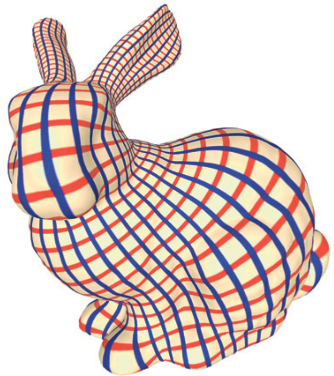
# Objectives

- Reconstitute analysis within *CAD geometry*
- *Simplify* analysis model development thereby
- *Integrate* design and analysis

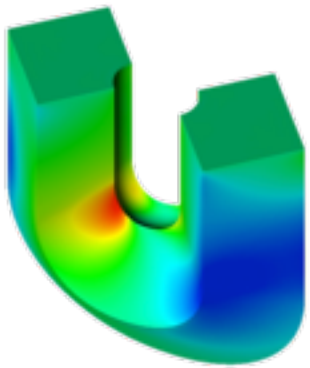
**Isogeometric Analysis**

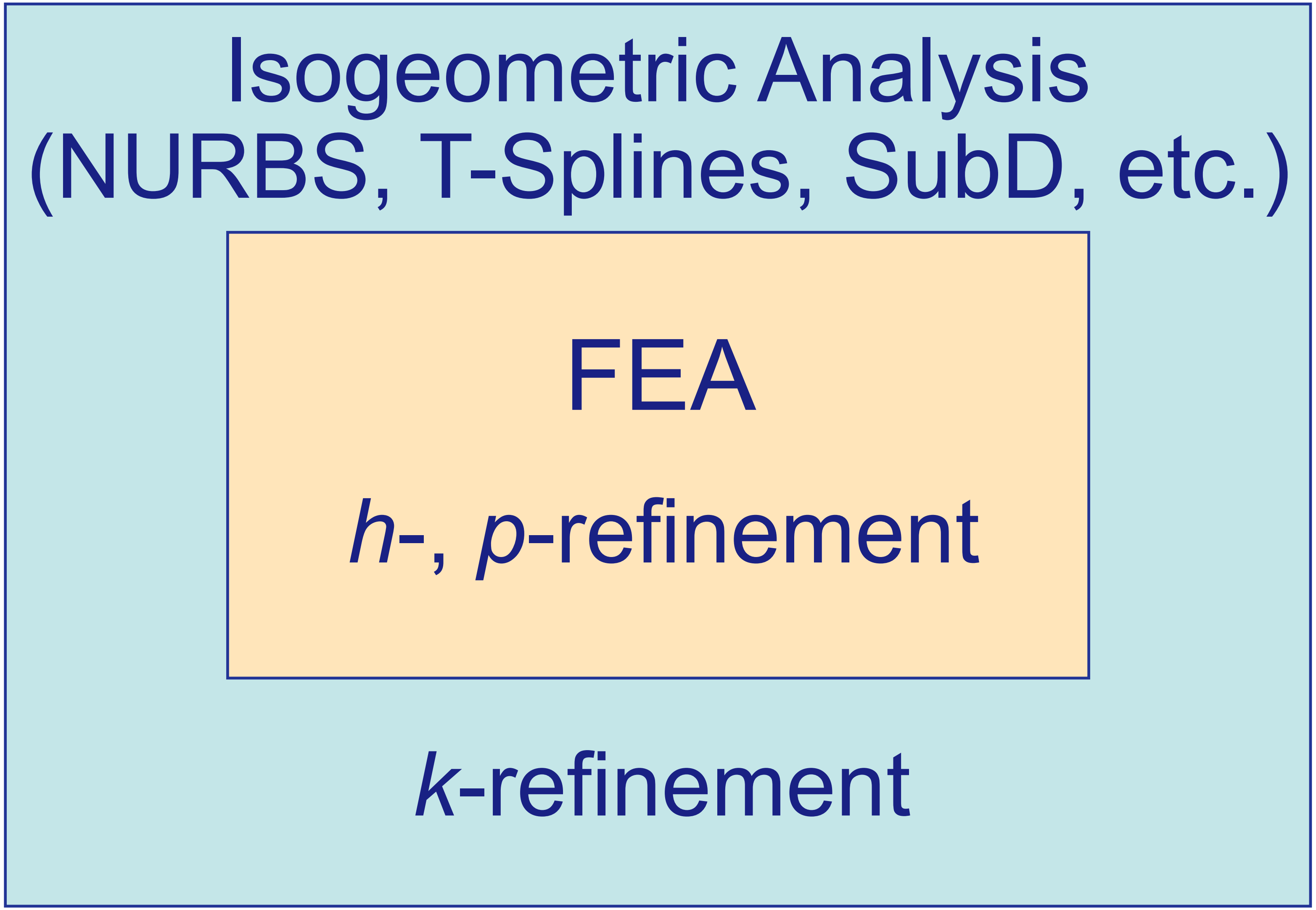
# Isogeometric Analysis

- Based on technologies (e.g., NURBS, T-splines, etc.) from *computational geometry* used in:
  - Design
  - Animation
  - Graphic art
  - Visualization



- Includes standard FEA as a special case, but offers other possibilities:
  - Precise and efficient geometric modeling
  - Simplified mesh refinement
  - Smooth basis functions with compact support
  - Superior approximation properties
  - *Integration* of design and analysis

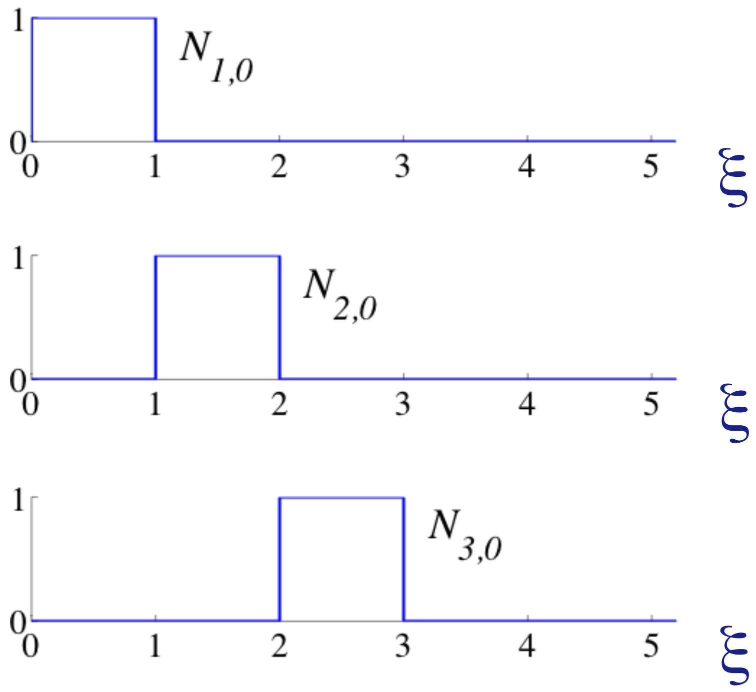


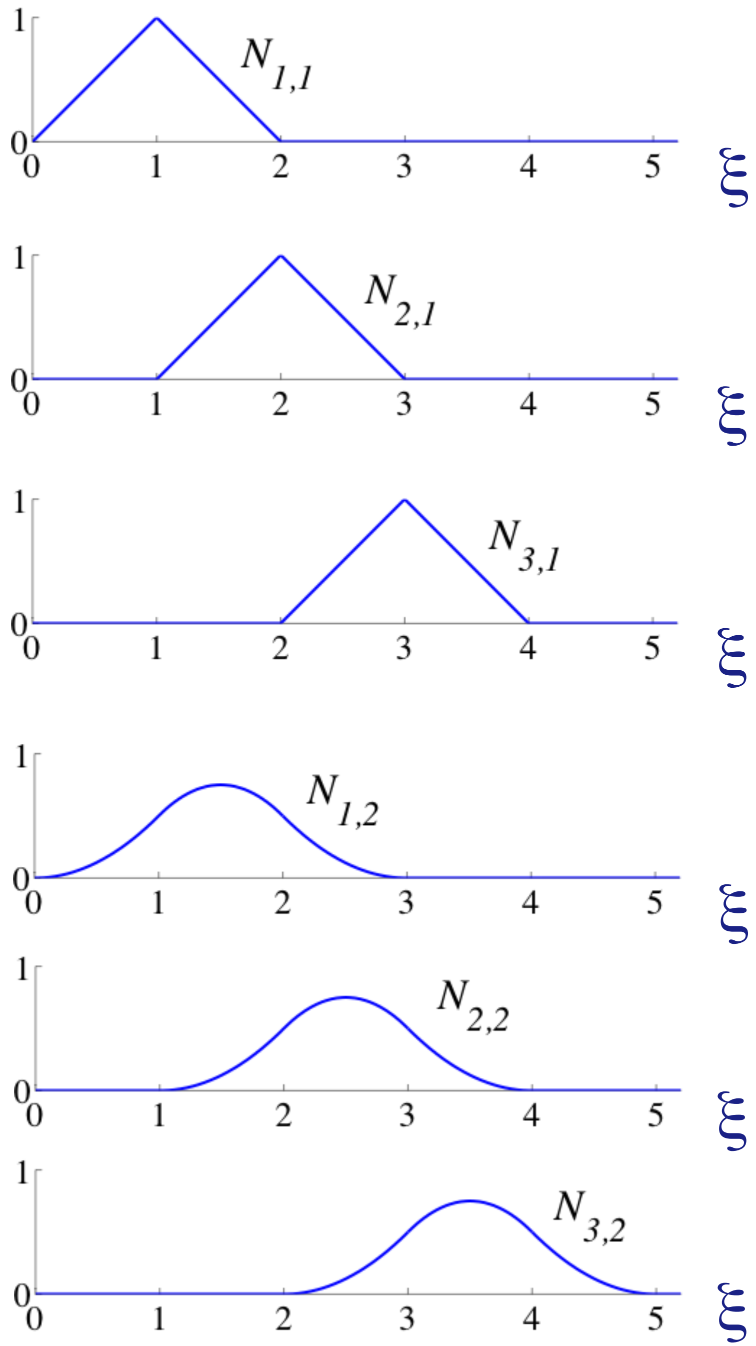

Isogeometric Analysis
(NURBS, T-Splines, SubD, etc.)
FEA
h-, p-refinement
k-refinement

# B-spline Basis Functions

$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \le \xi < \xi_{i+1}, \\ 0 & \text{otherwise} \end{cases}$$

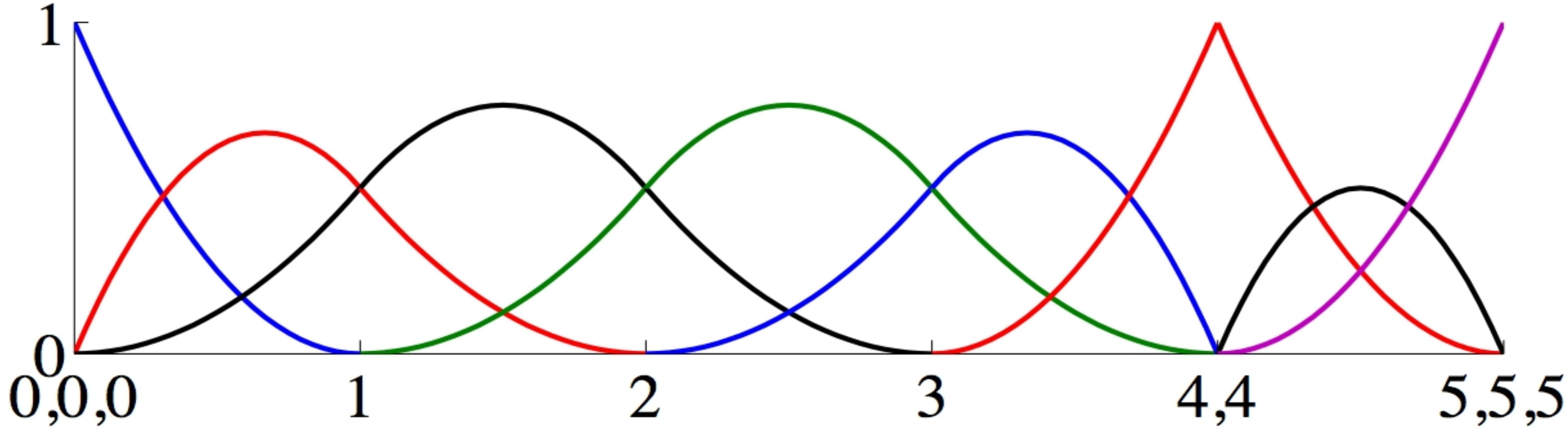
$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) \; + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$




B-spline basis functions of order 0, 1, 2 for a *uniform knot vector:*

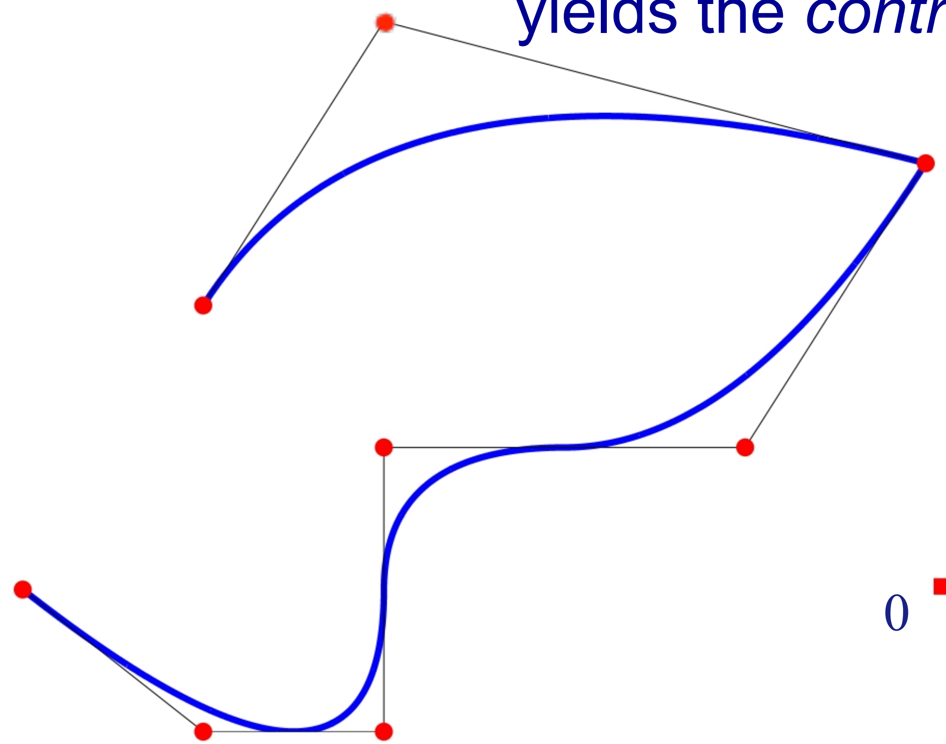
$$\Xi = \{0,1,2,3,4,\ldots\}$$

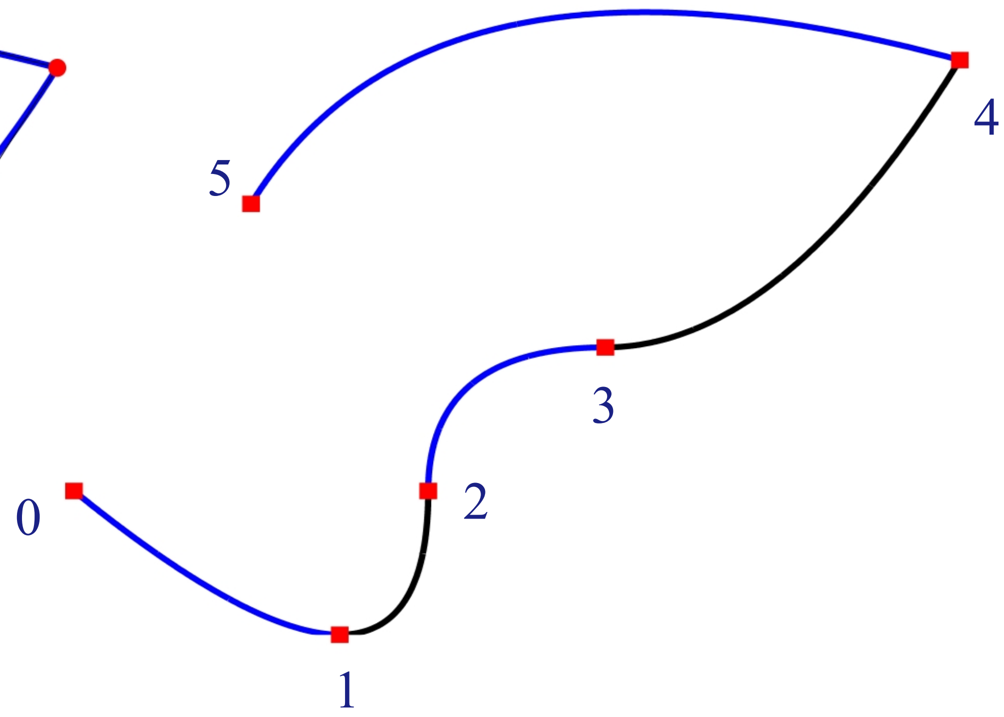



Quadratic ($p$=2) basis functions for an
*open, non-uniform knot vector:*

$$\Xi = \{0,0,0,1,2,3,4,4,5,5,5\}$$

# Linear interpolation of control points yields the *control polygon*

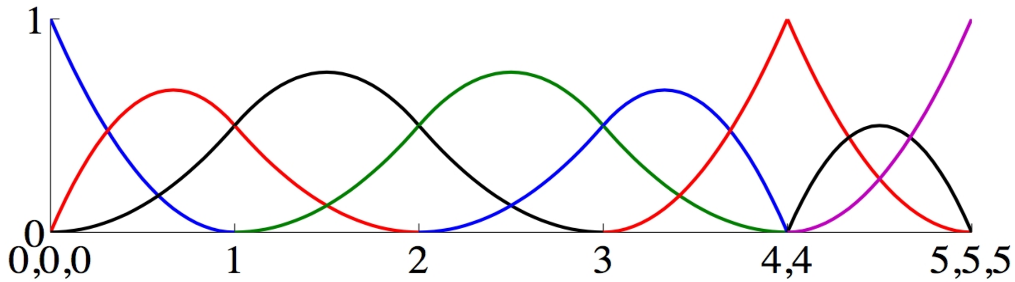




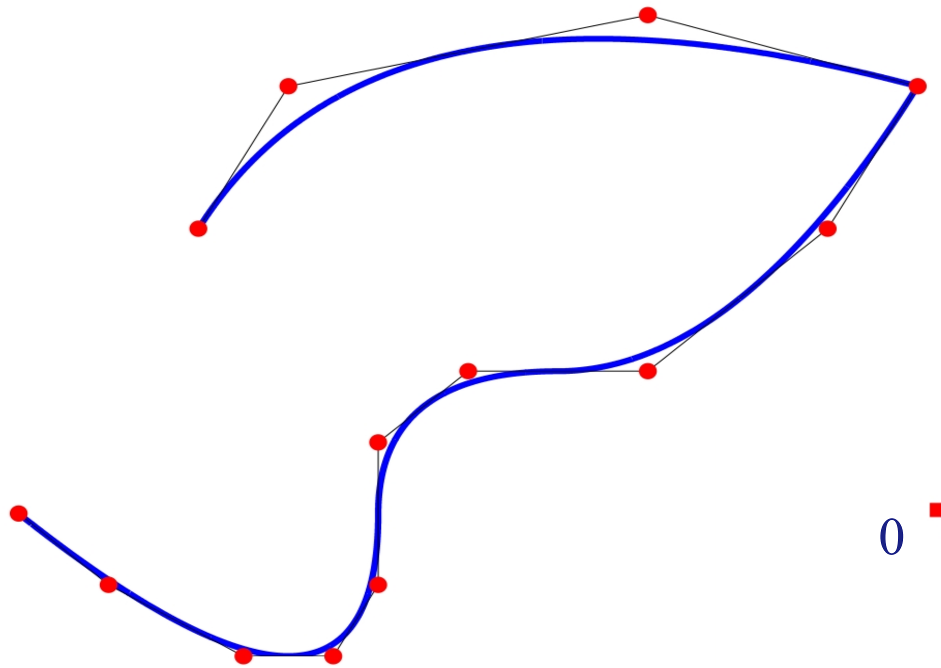

● - control points

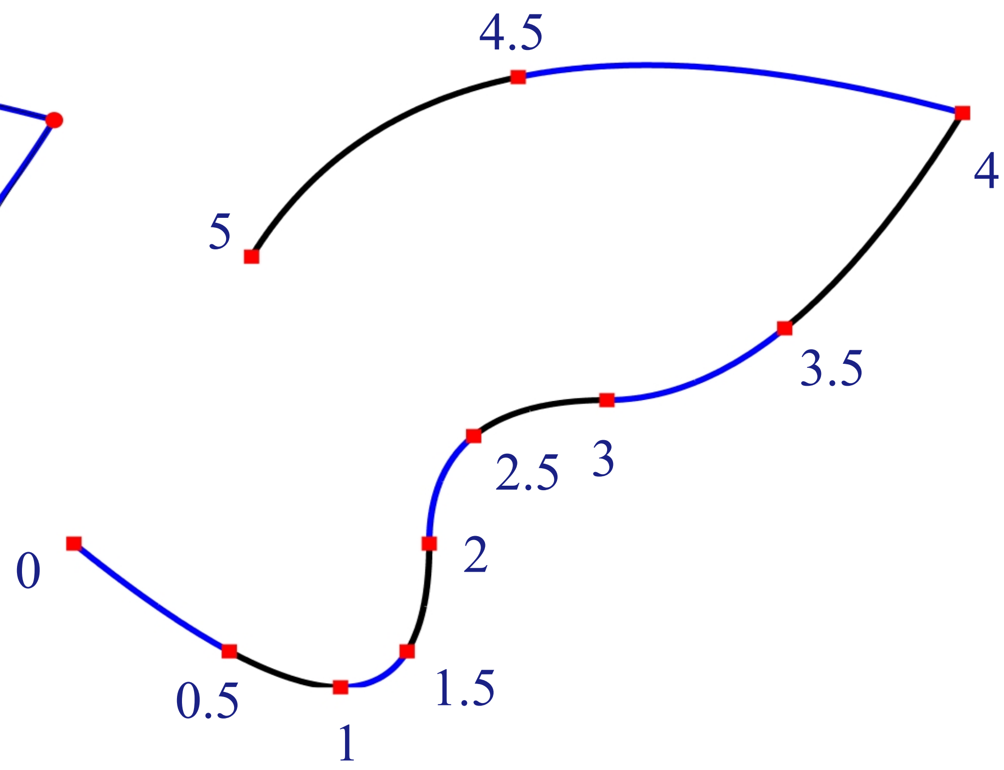
■ - knots

Quadratic basis



# *h*-refined Curve

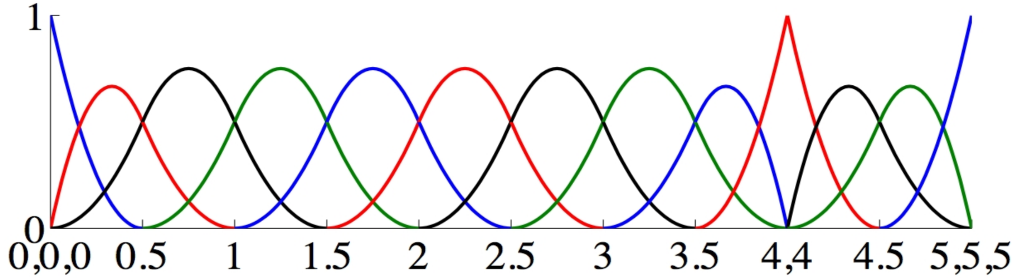




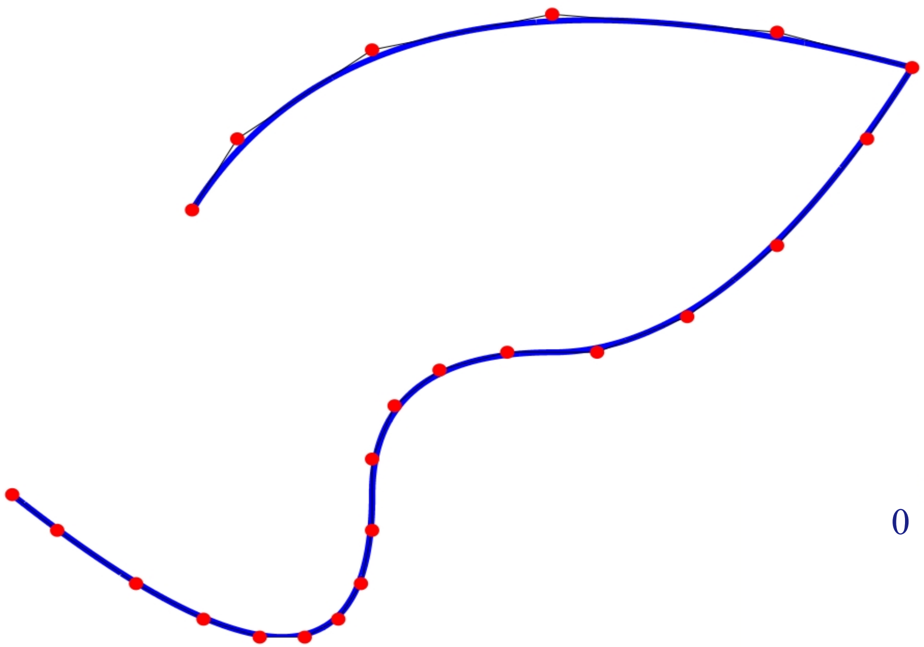

● - control points

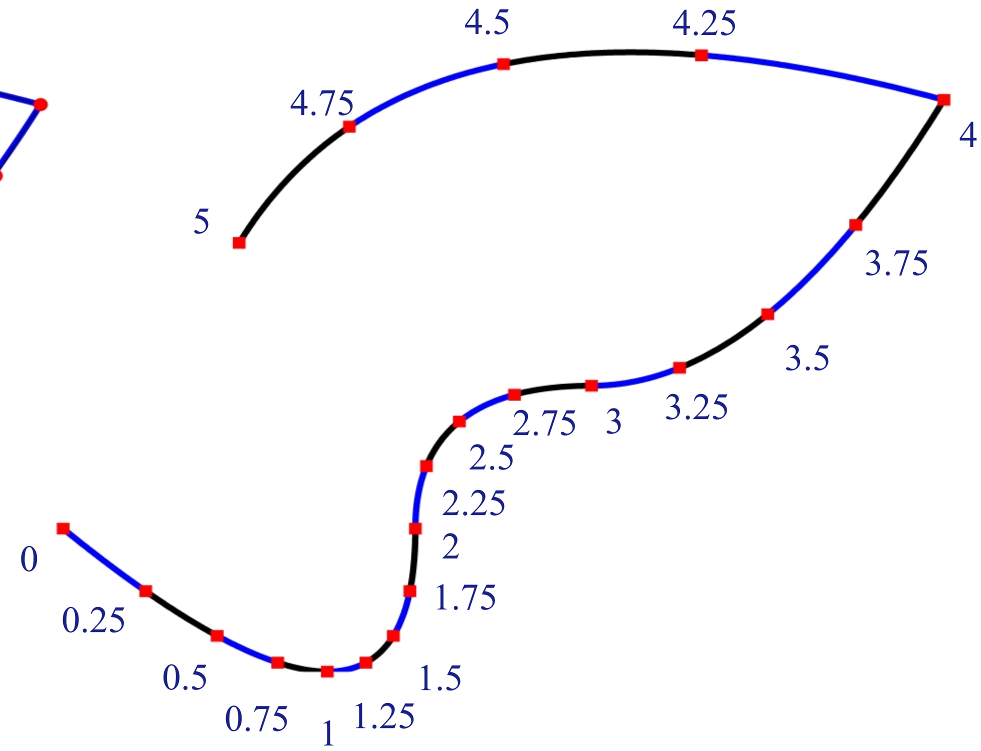
■ - knots

Quadratic basis



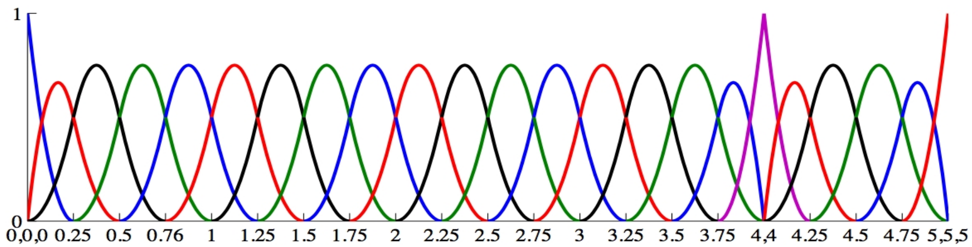
# Further *h*-refined Curve



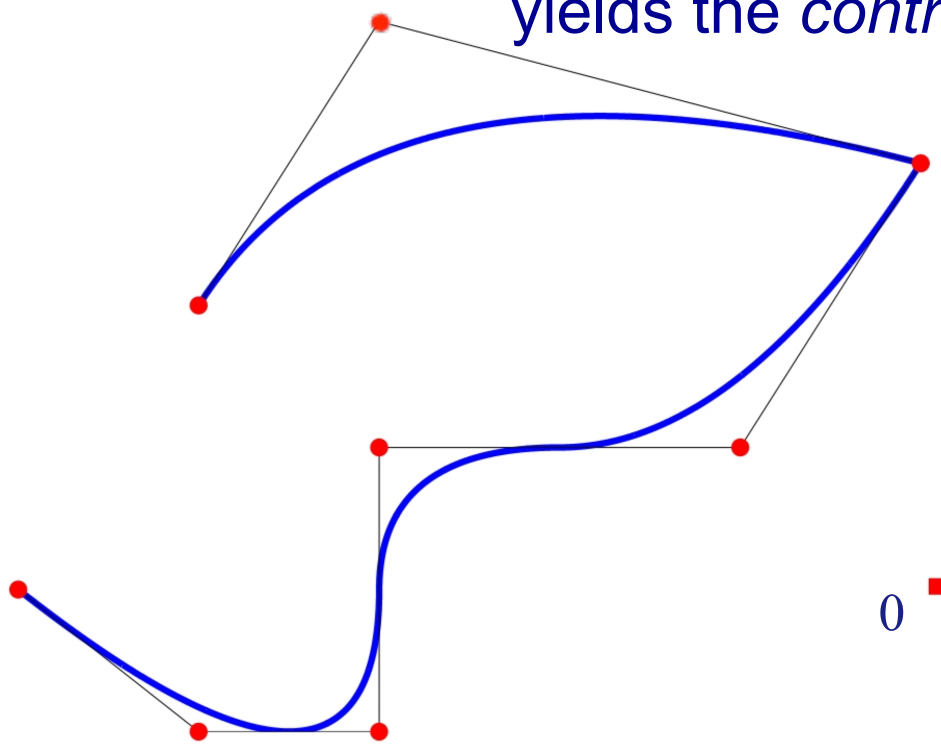


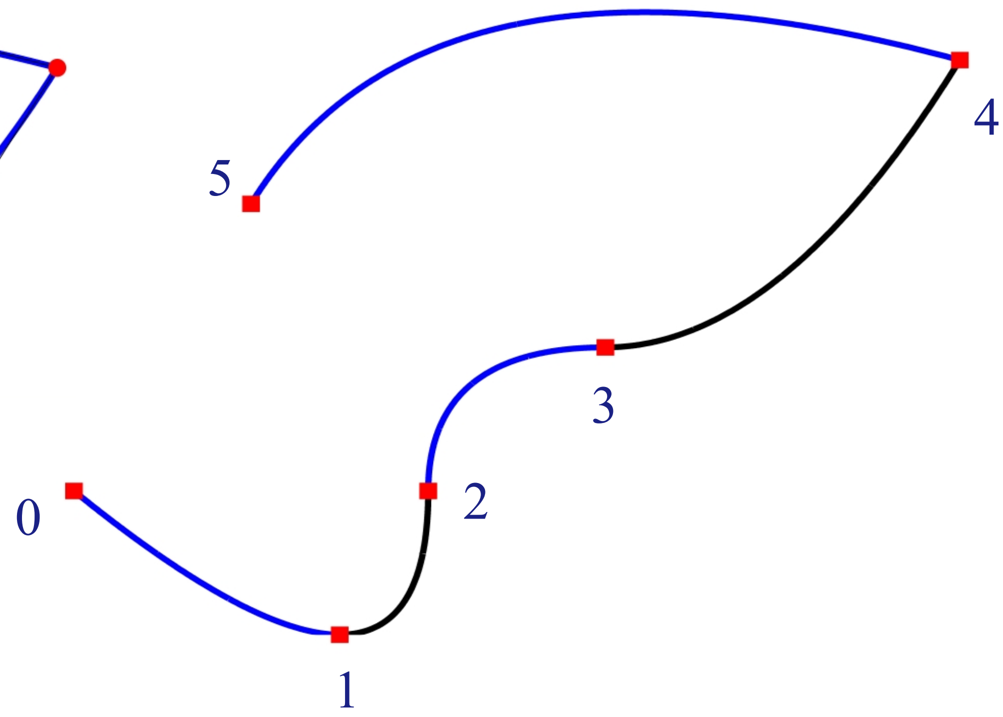

● - control points ■ - knots

Quadratic basis



Linear interpolation of control points yields the *control polygon*

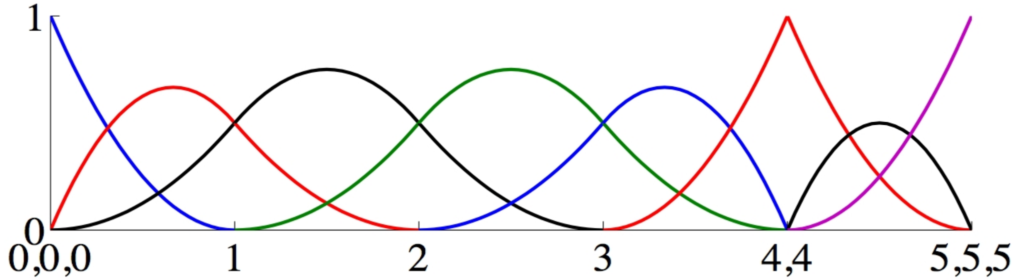




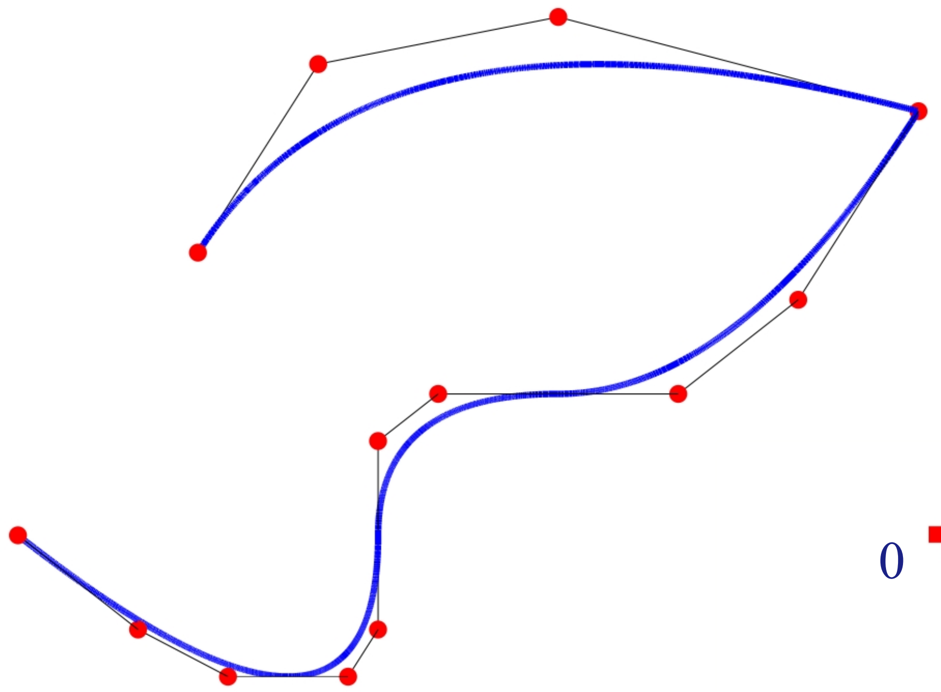

● - control points

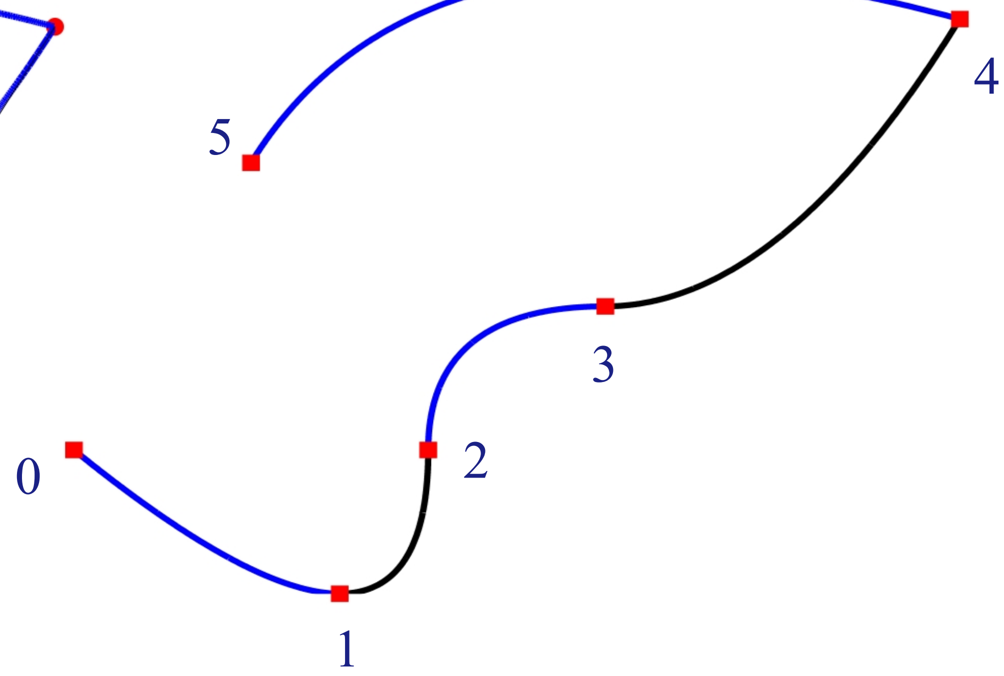
■ - knots

Quadratic basis



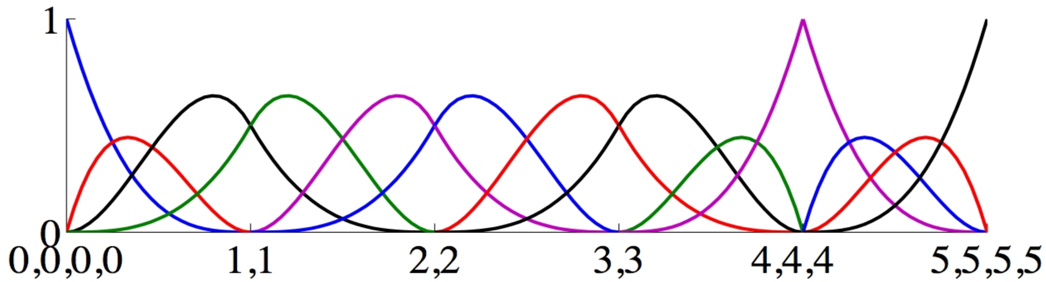
# Cubic *p*-refined Curve



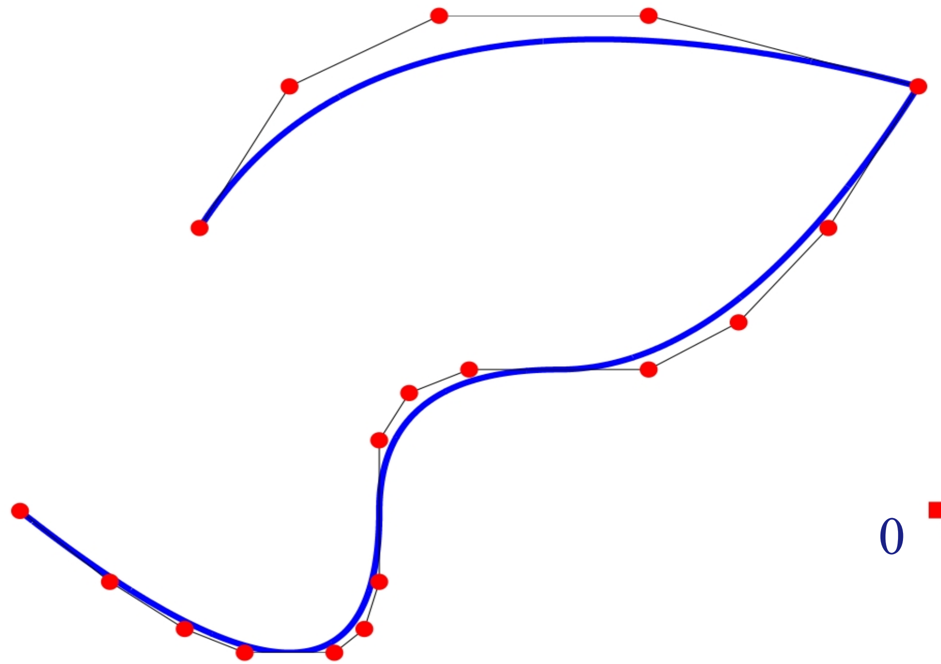


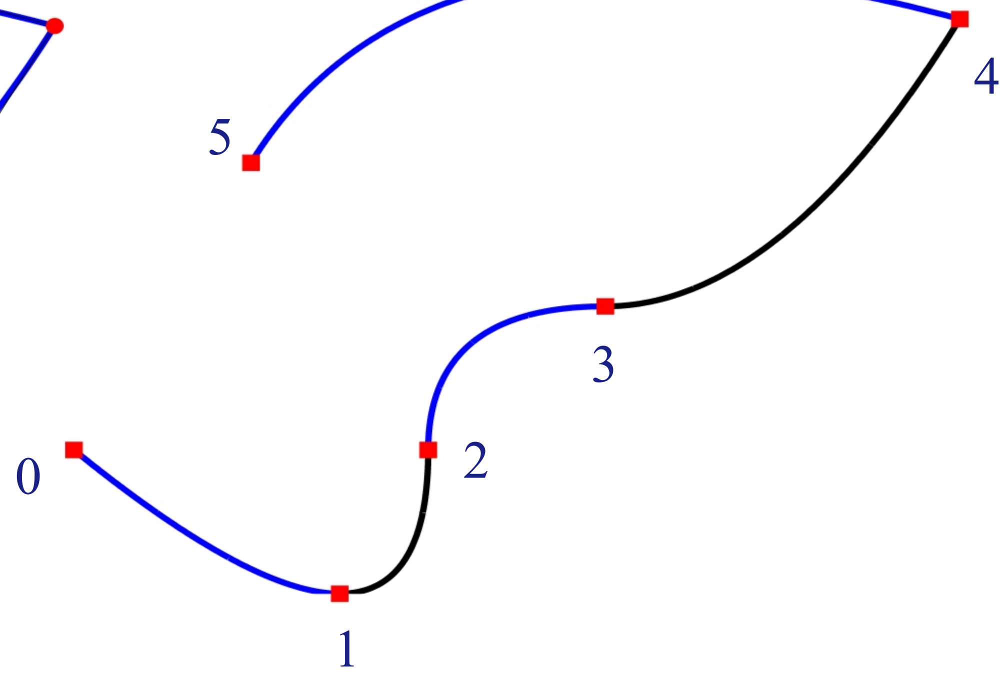

● - control points ■ - knots

Cubic basis



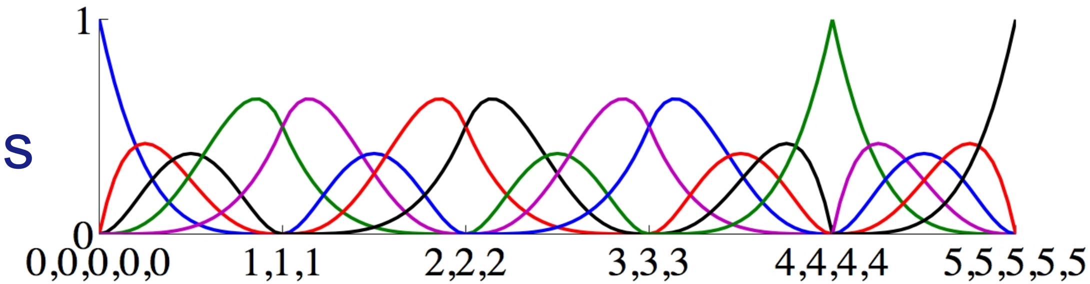
# Quartic *p*-refined Curve





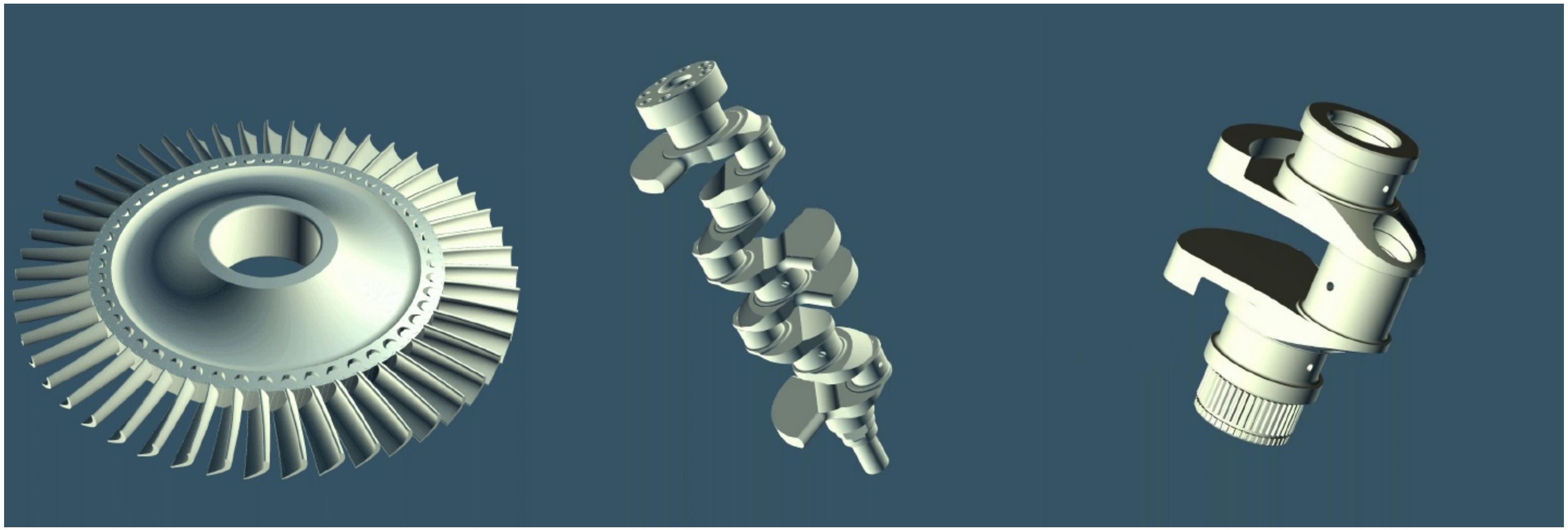

● - control points ■ - knots

Quartic basis

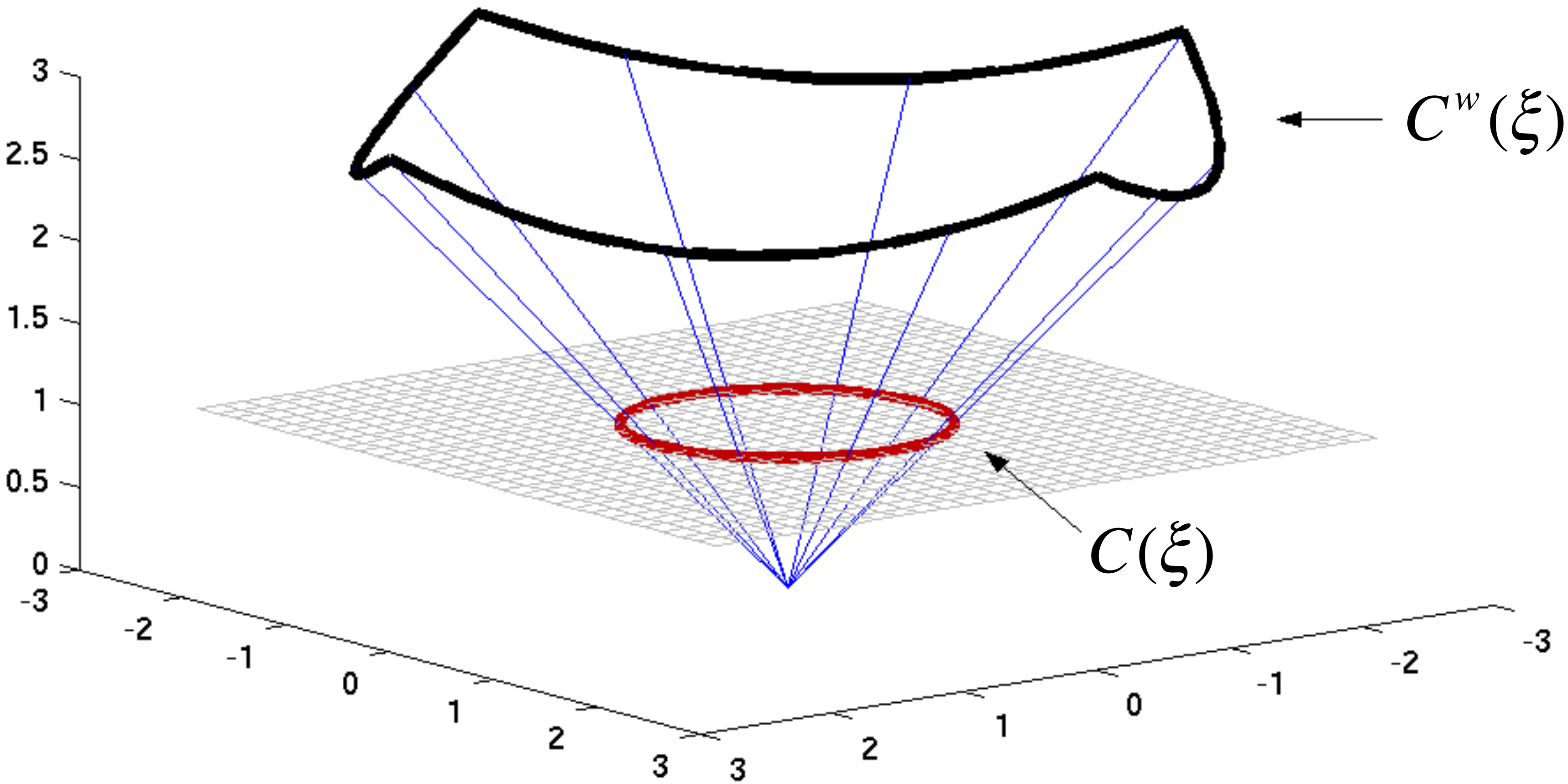


# Non-Uniform Rational B-Splines

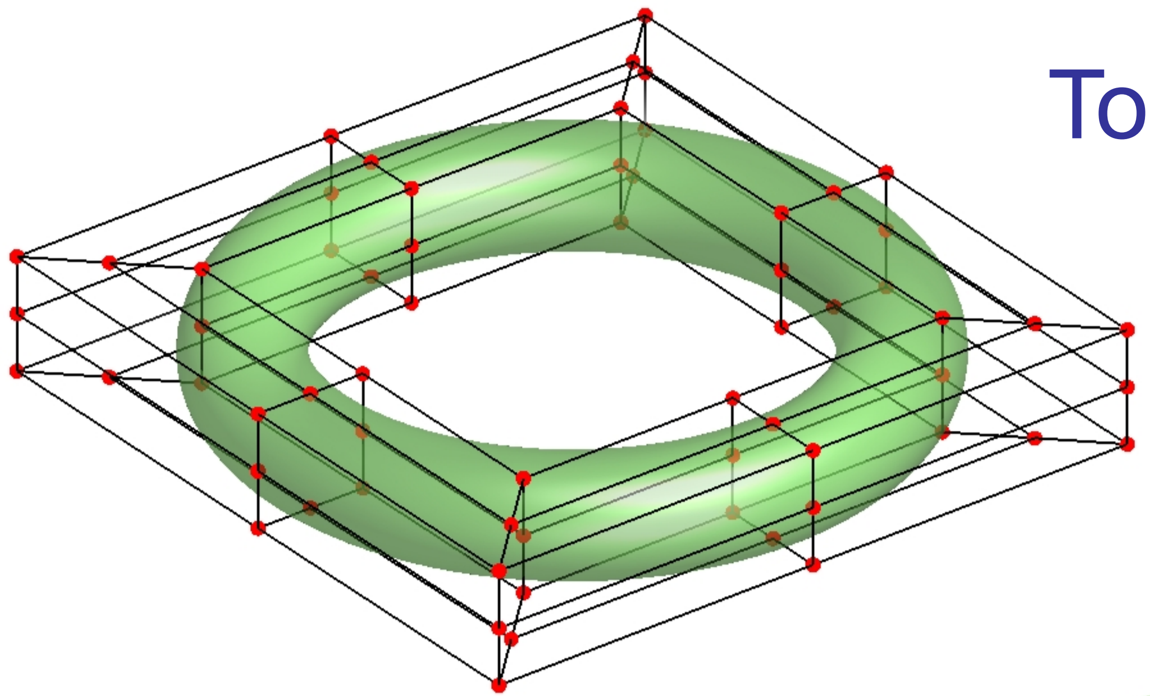
- NURBS are the most commonly used computer aided geometric design (CAGD) technology in engineering



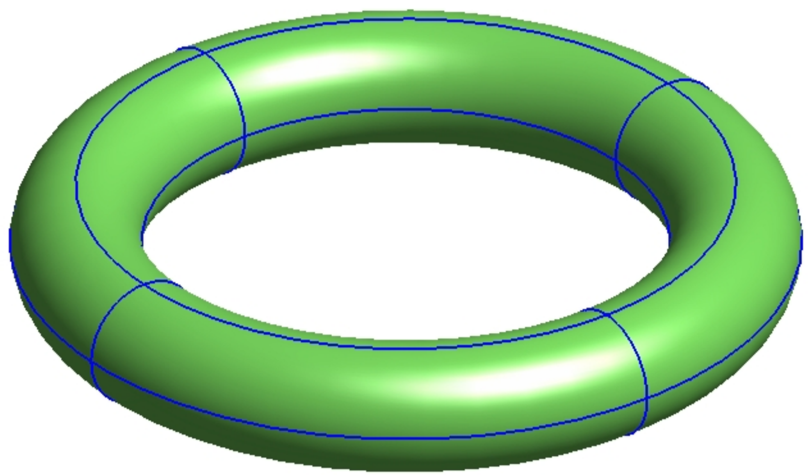
# Circle from 3D Piecewise Quadratic Curves



# Toroidal Surface

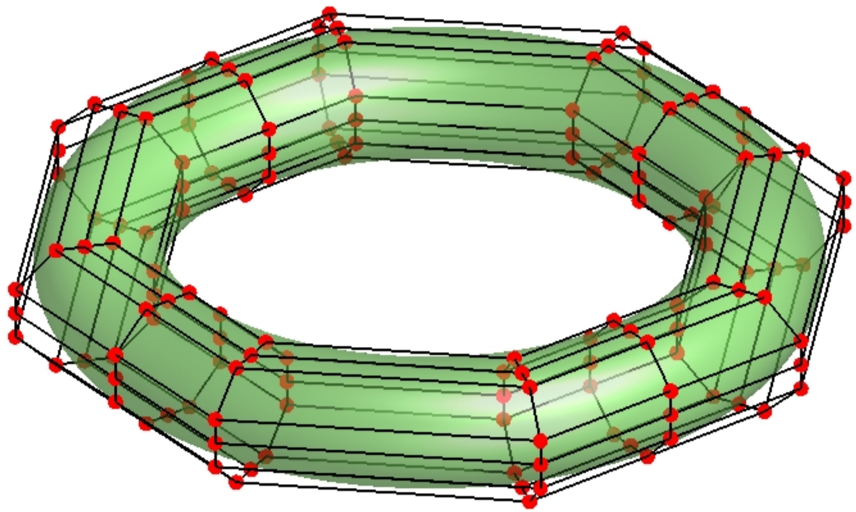


Control net

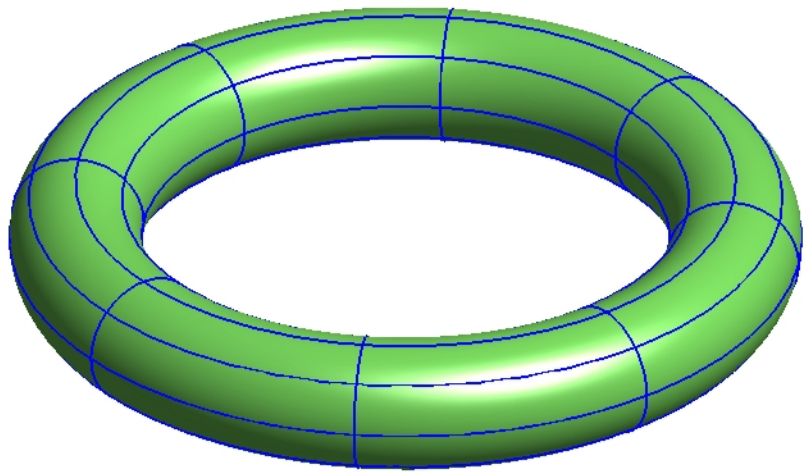


Mesh

# *h*-refined Surface

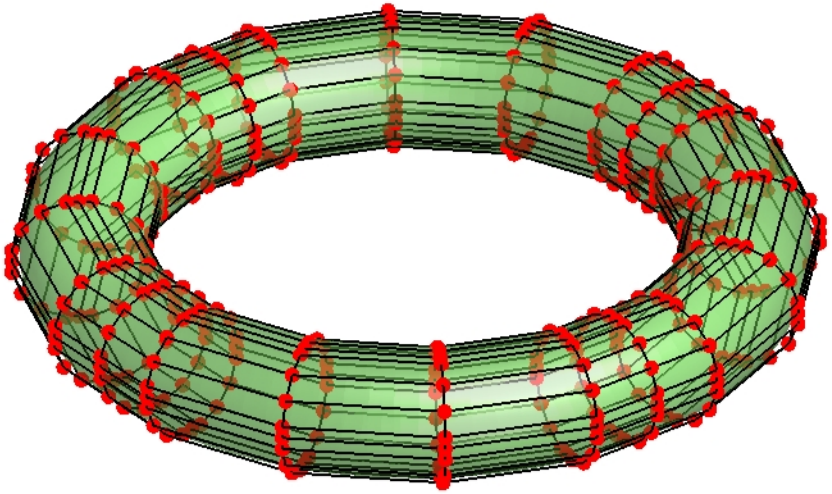


Control net

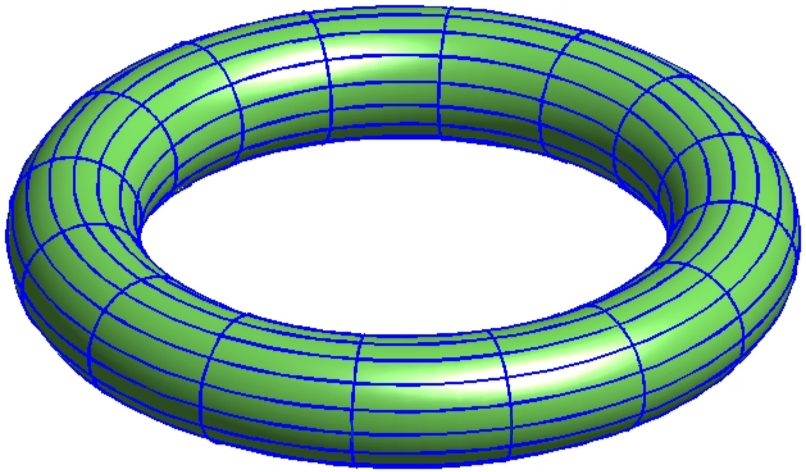


Mesh

# Further *h*-refined Surface



Control net



Mesh

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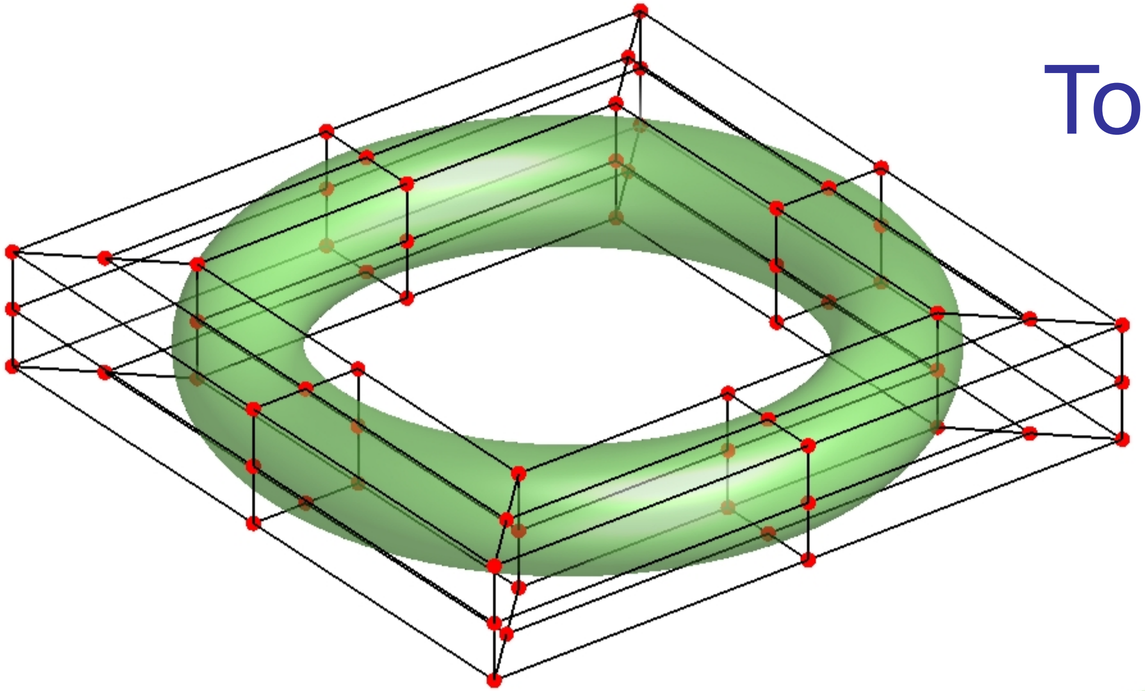

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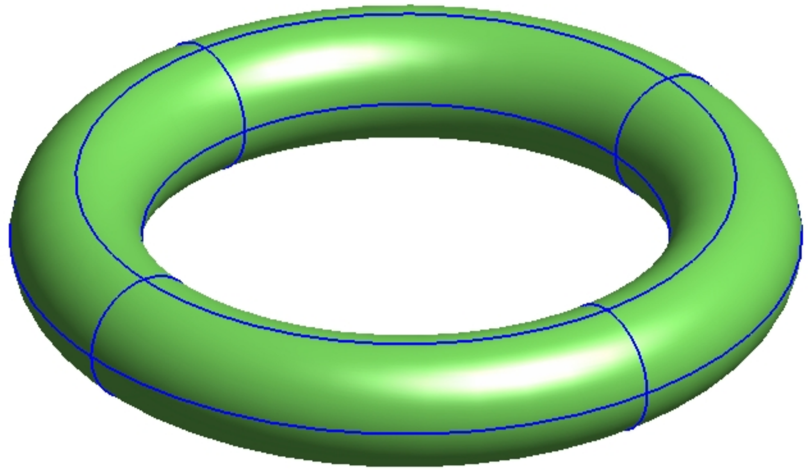
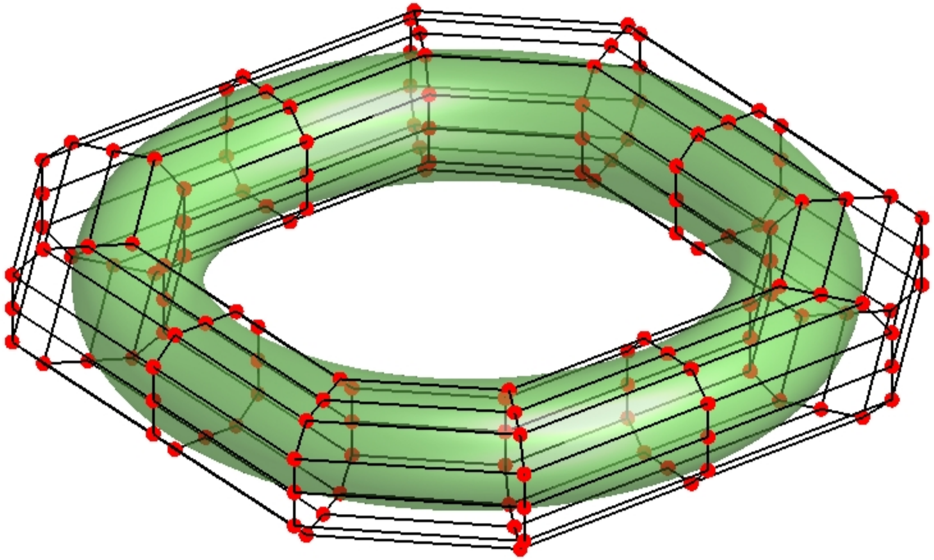
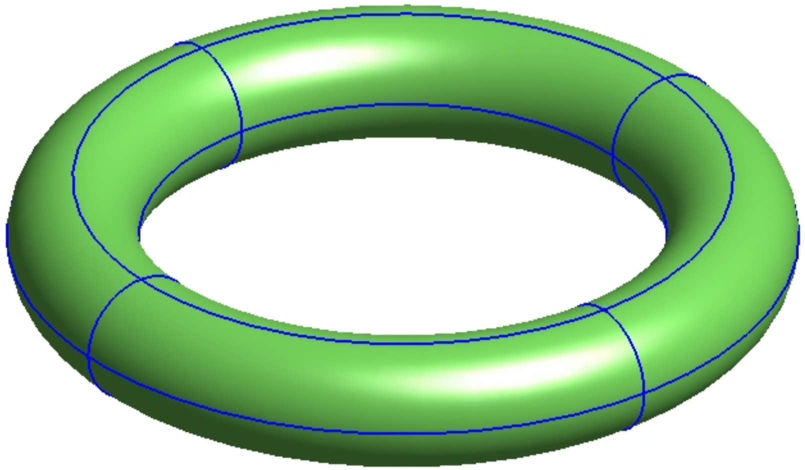


Mesh

# Cubic *p*-refined Surface

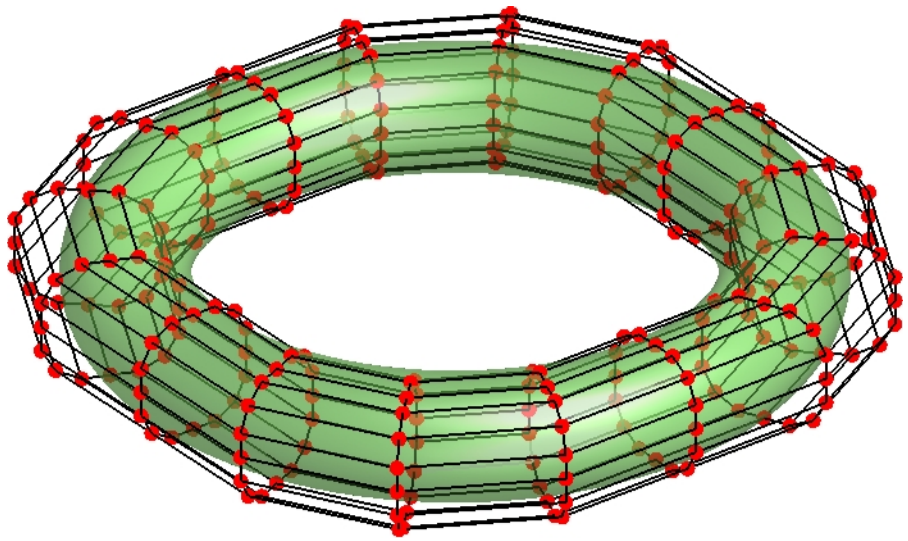


Control net

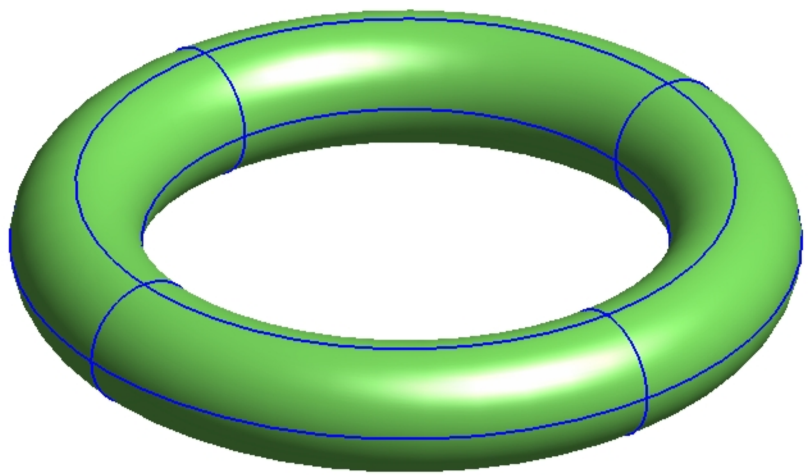


Mesh

# Quartic *p*-refined Surface

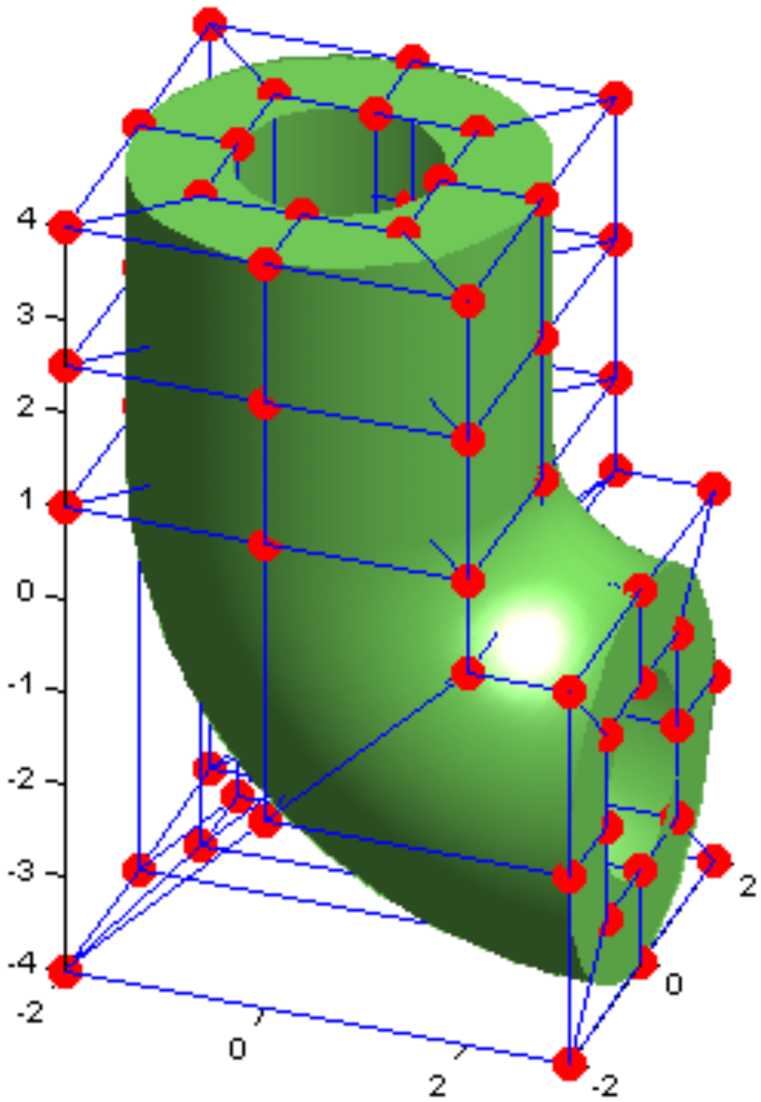


Control net

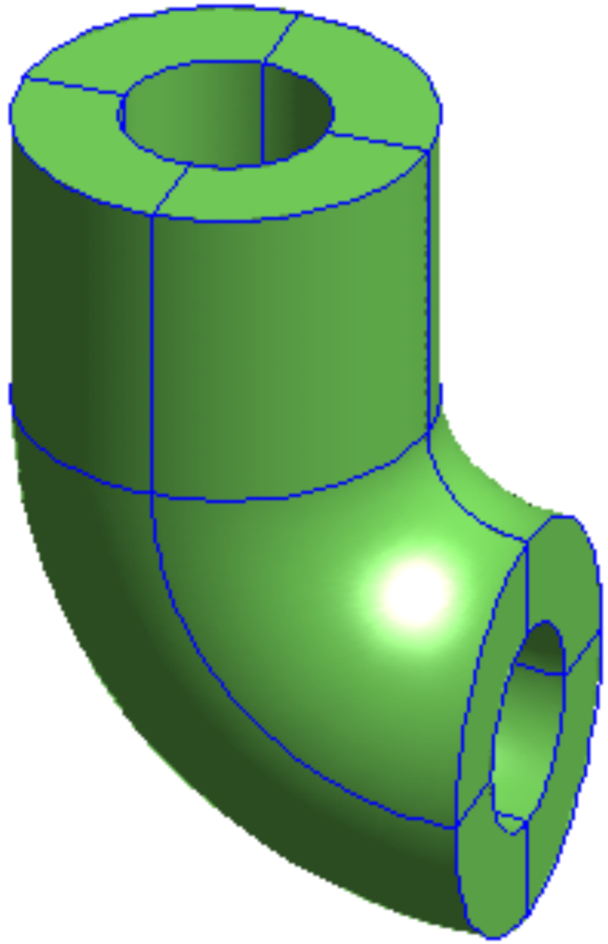


Mesh

## Control Net




## Mesh



# Finite Element Analysis and NURBS-based Isogeometric Analysis

| |
|---|
| ▪ Compact support |
| ▪ Partition of unity |
| ▪ Affine covariance |
| ▪ Isoparametric concept |
| ▪ Patch tests satisfied |
| ▪ Error estimates in Sobolev norms* |

*Y. Bazilevs, L. Beirão da Veiga, J.A. Cottrell, TJRH, & G. Sangalli, 2006

# An Examination of the Helmholtz Pollution Effect for FEM and NURBS

# Problem Statement

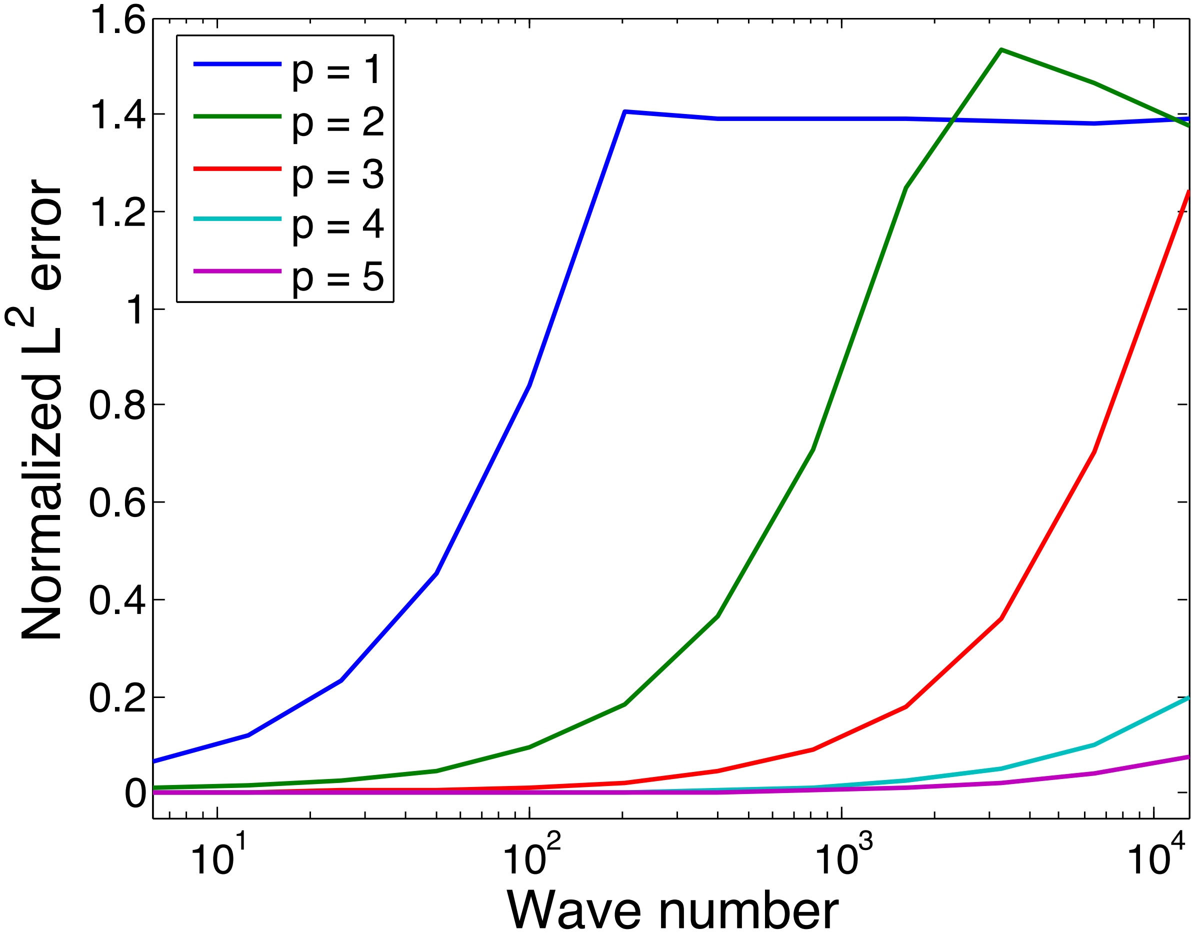
Model Problem:

$$u''(x) + k^2 u(x) = 0 \text{ on } (0,1)$$
$$u(0) = 1$$
$$u'(1) - iku(1) = 0$$

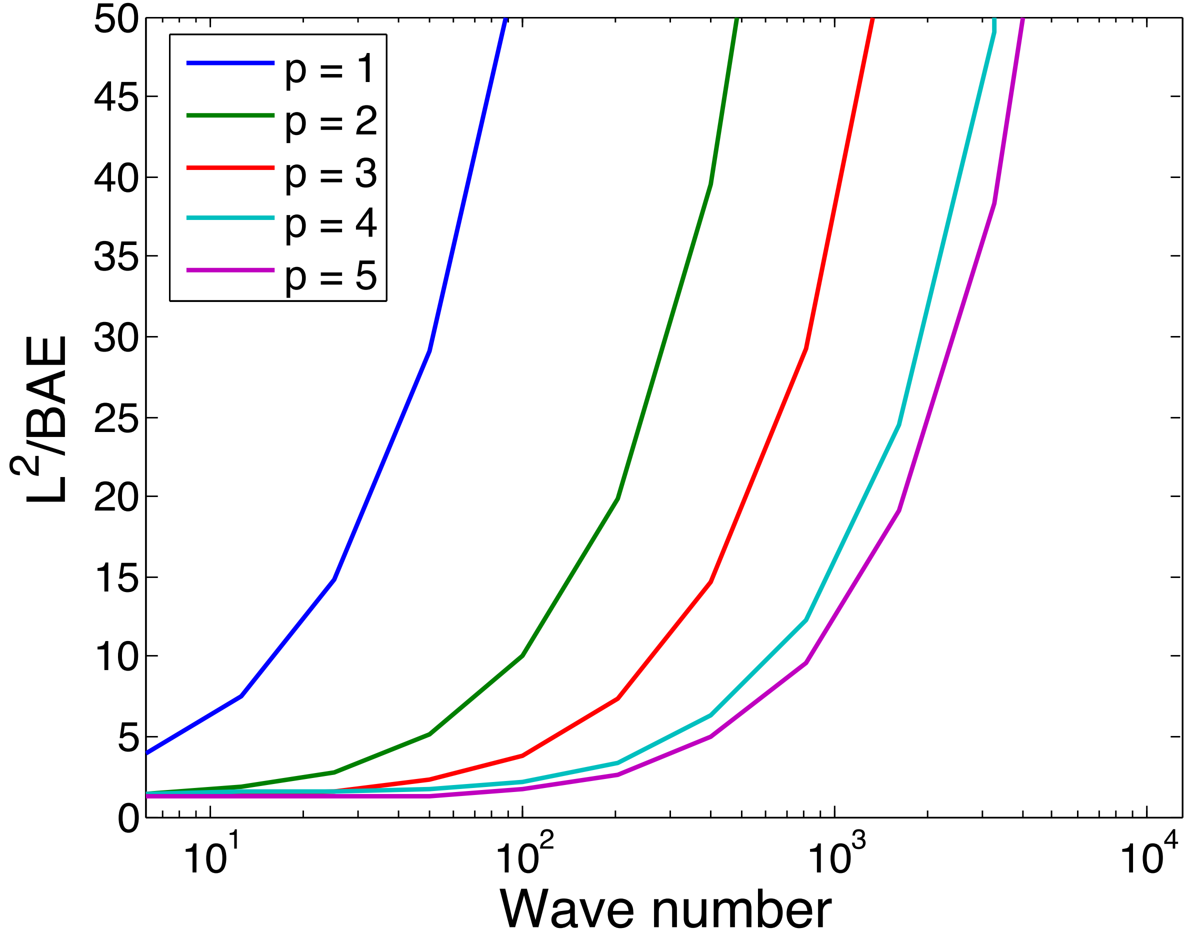
Exact Solution:

$$u(x) = \exp(ikx)$$

# Pollution: FEM

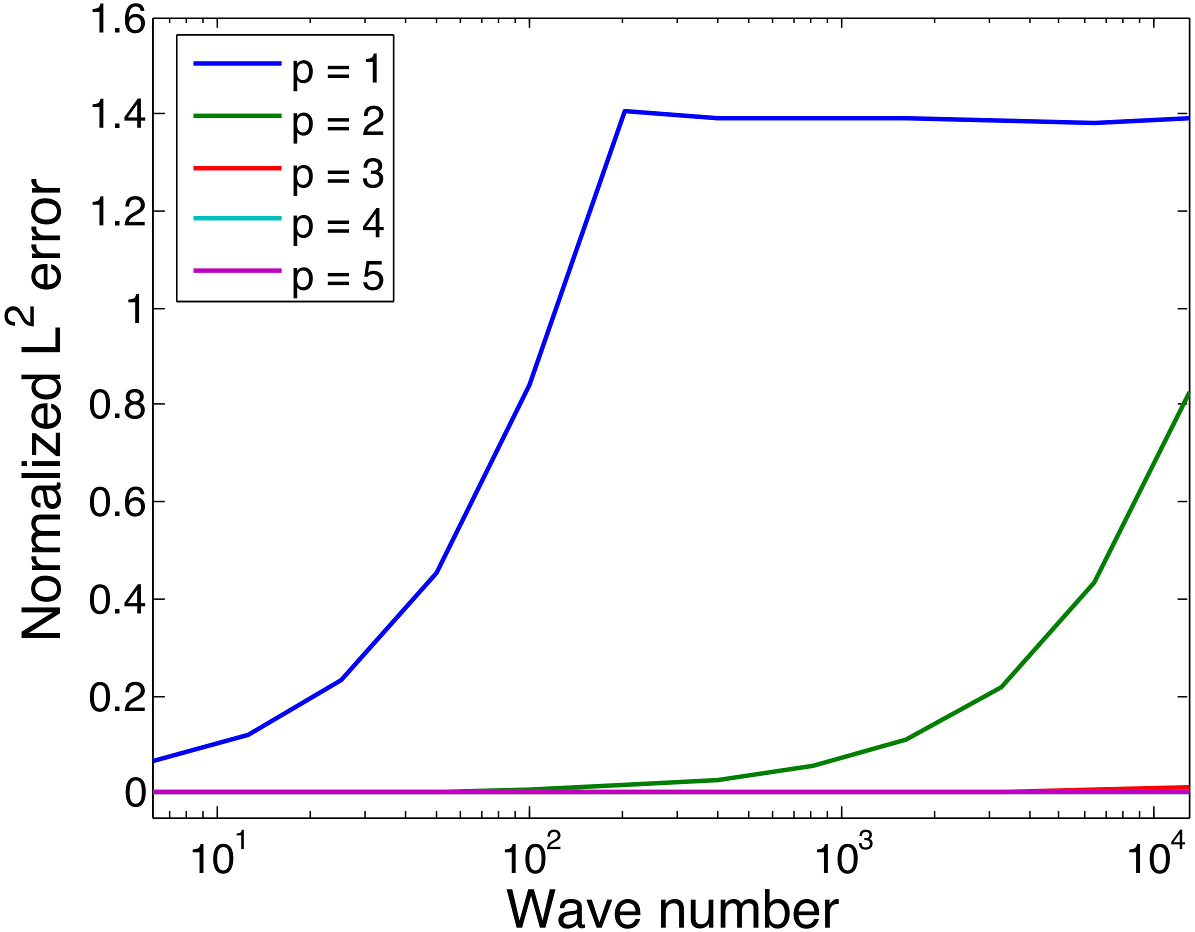


# Pollution: FEM




BAE: Best Approximation Error

# Pollution: NURBS



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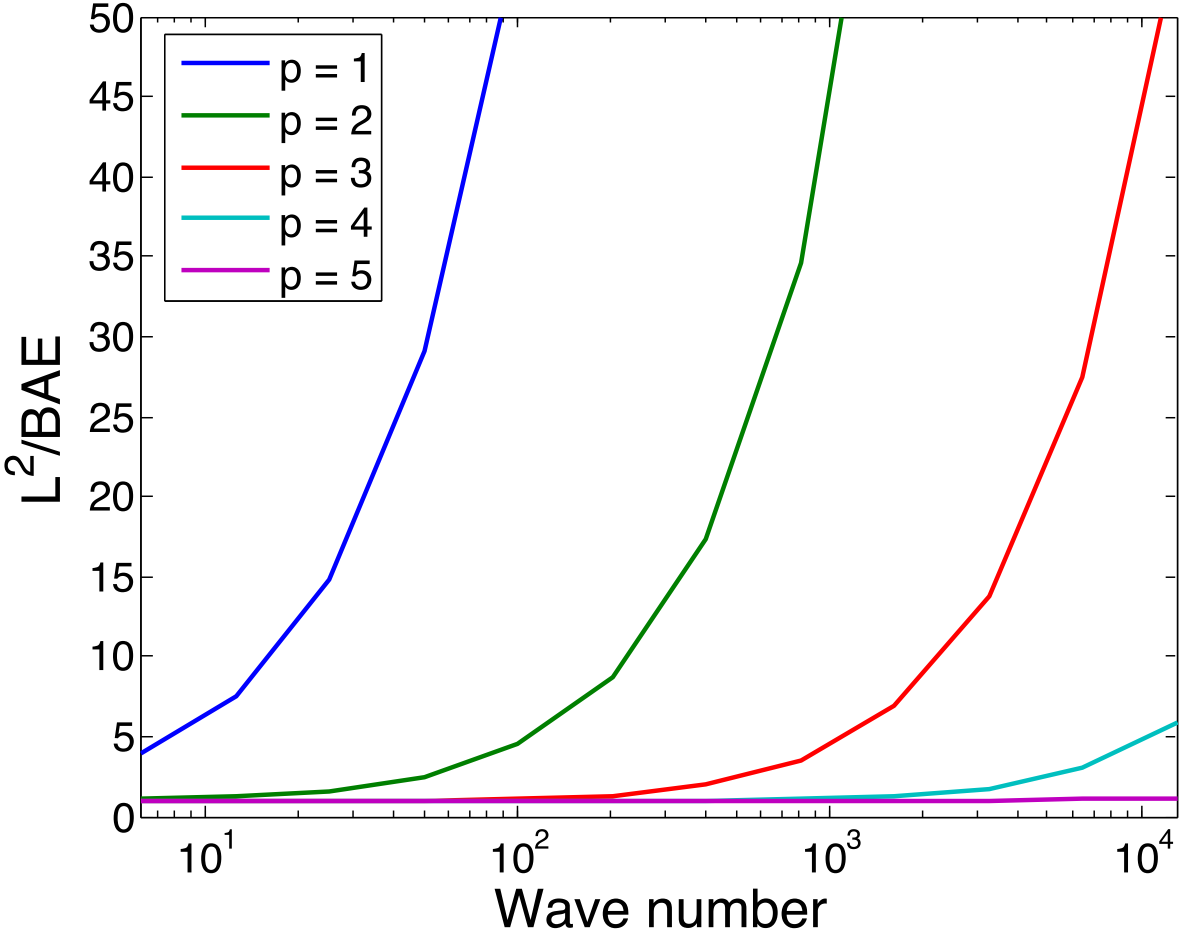
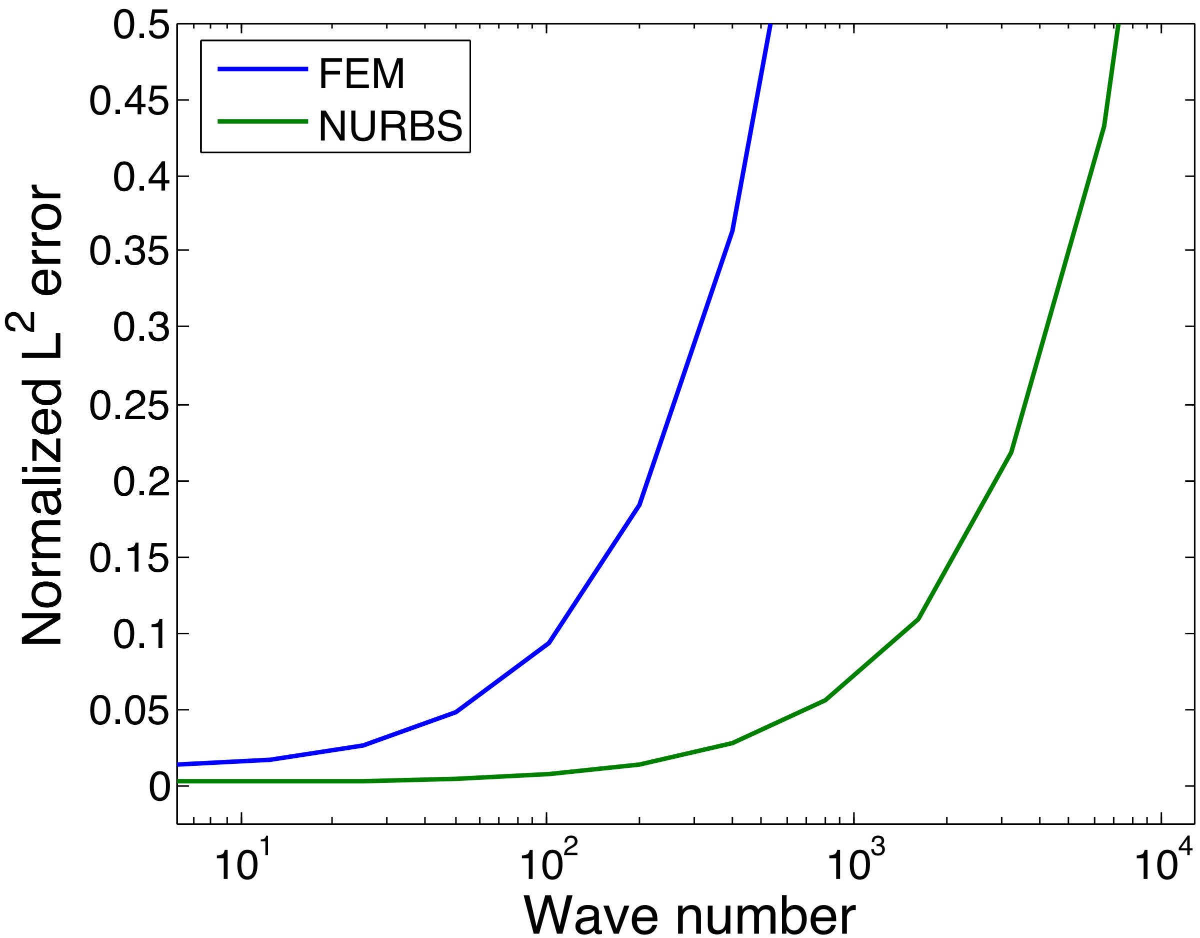
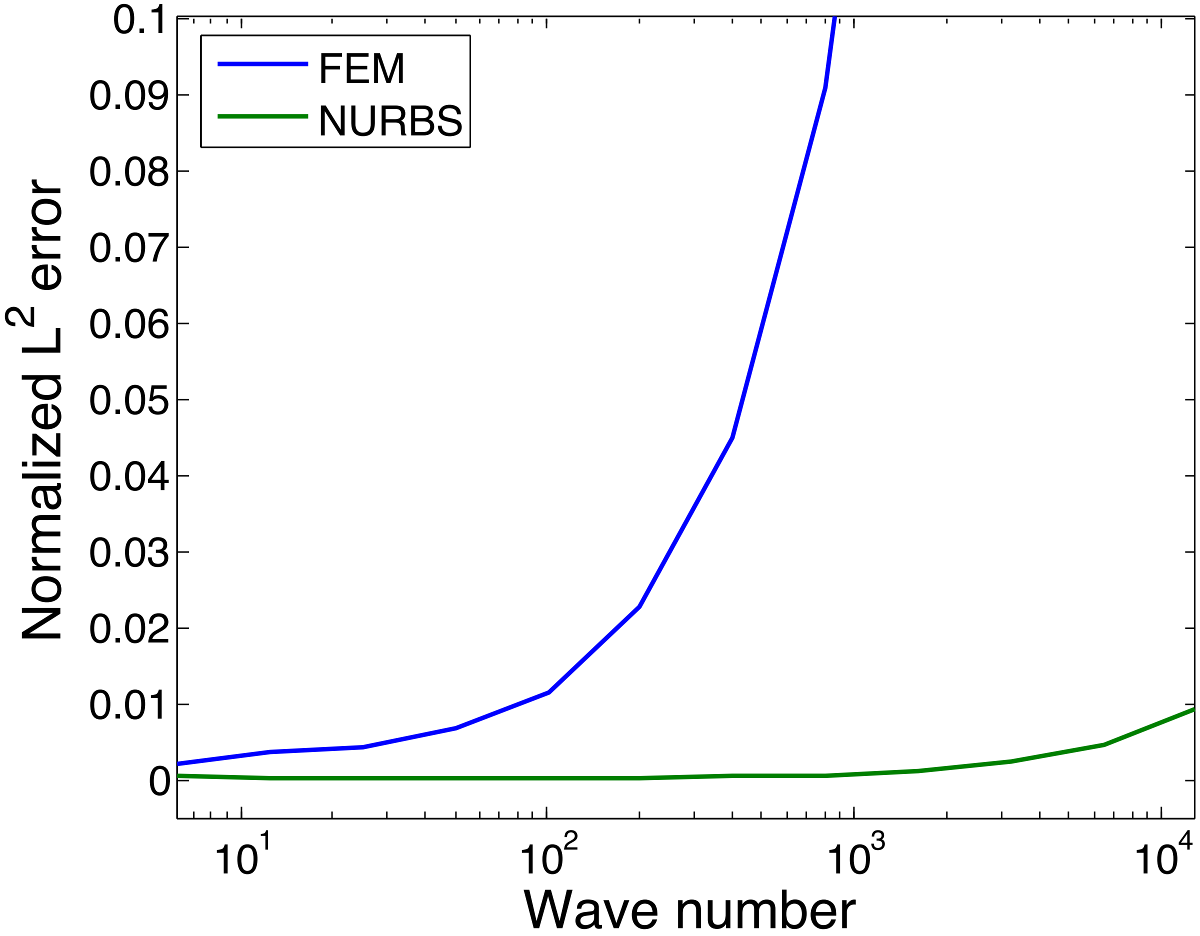



BAE: Best Approximation Error

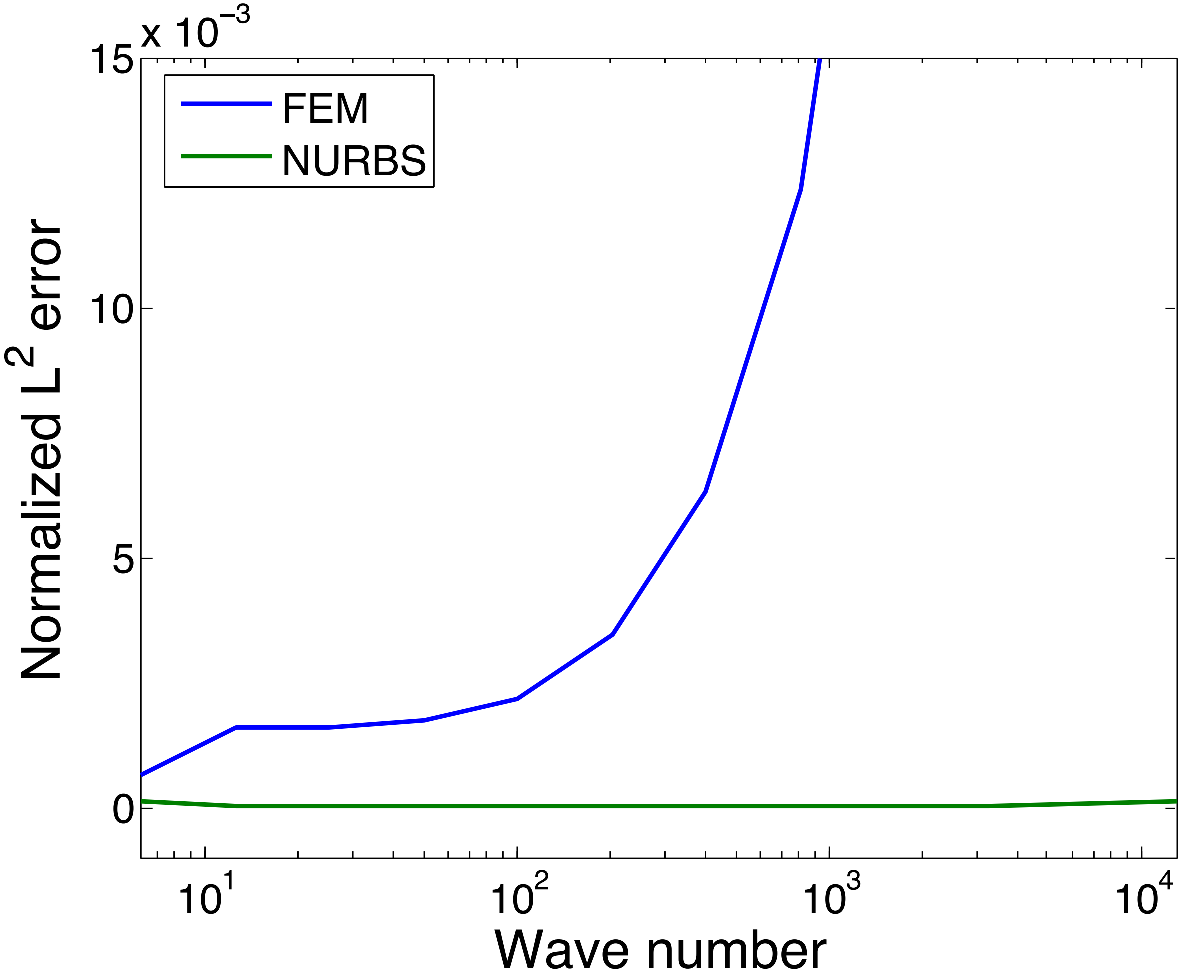
# Pollution: Degree 2 Comparison



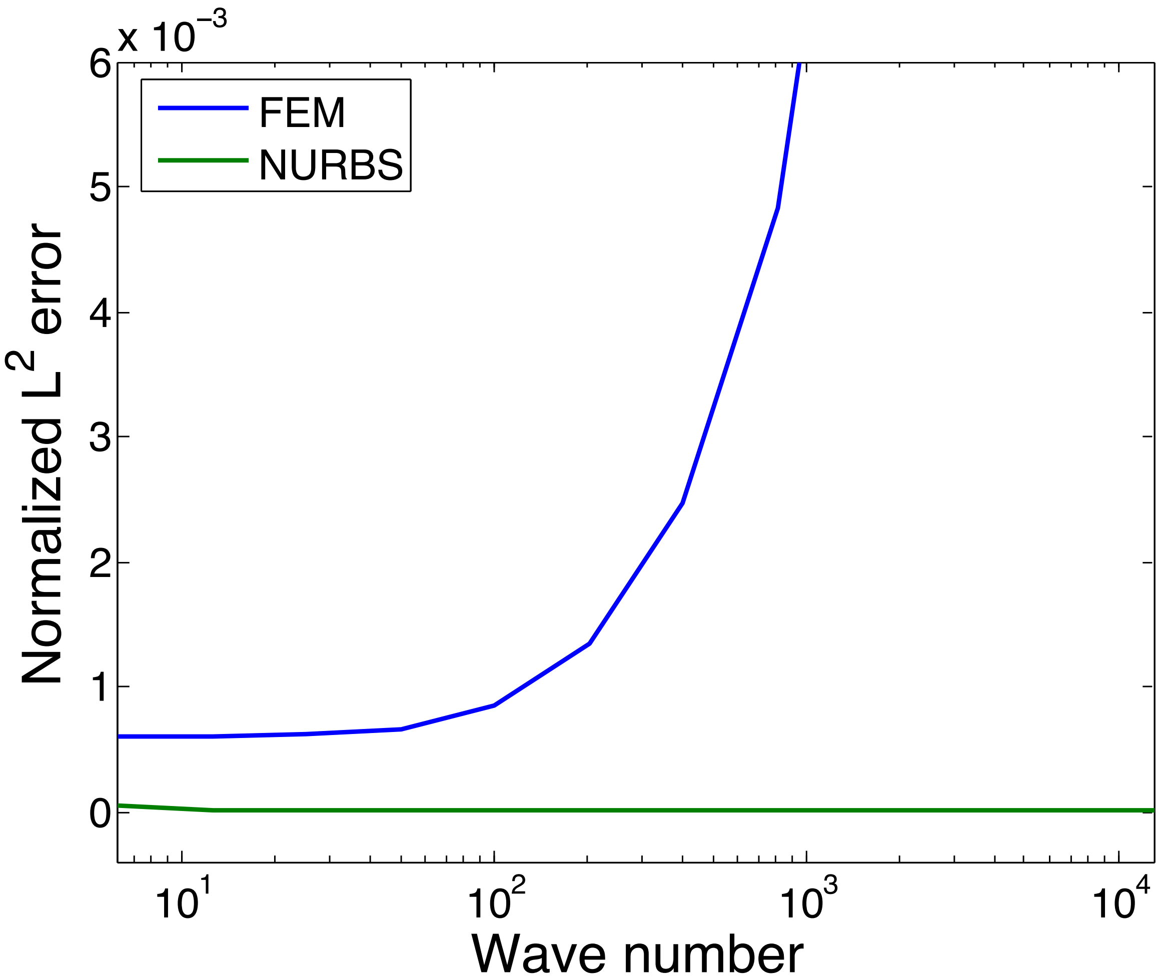
# Pollution: Degree 3 Comparison



# Pollution: Degree 4 Comparison

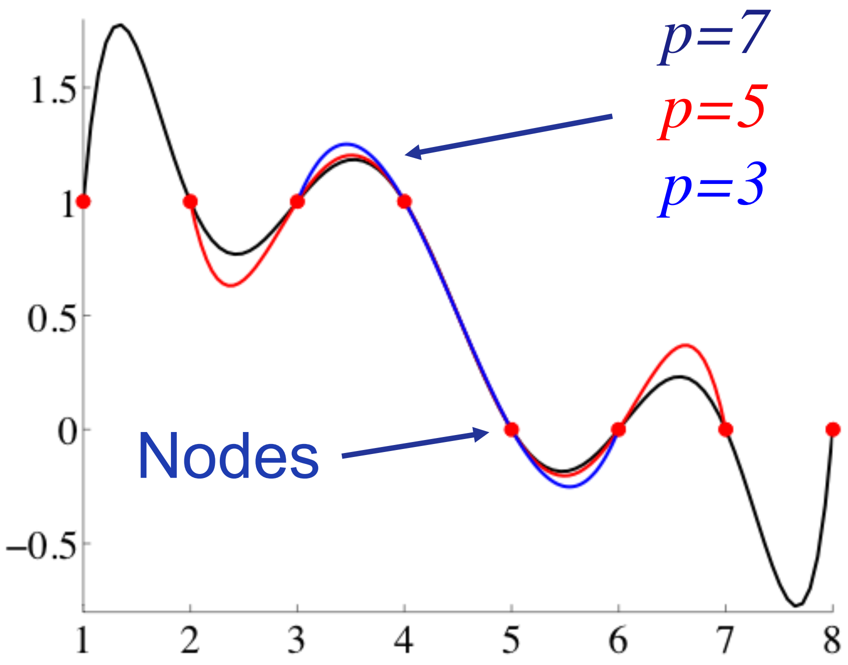


# Pollution: Degree 5 Comparison

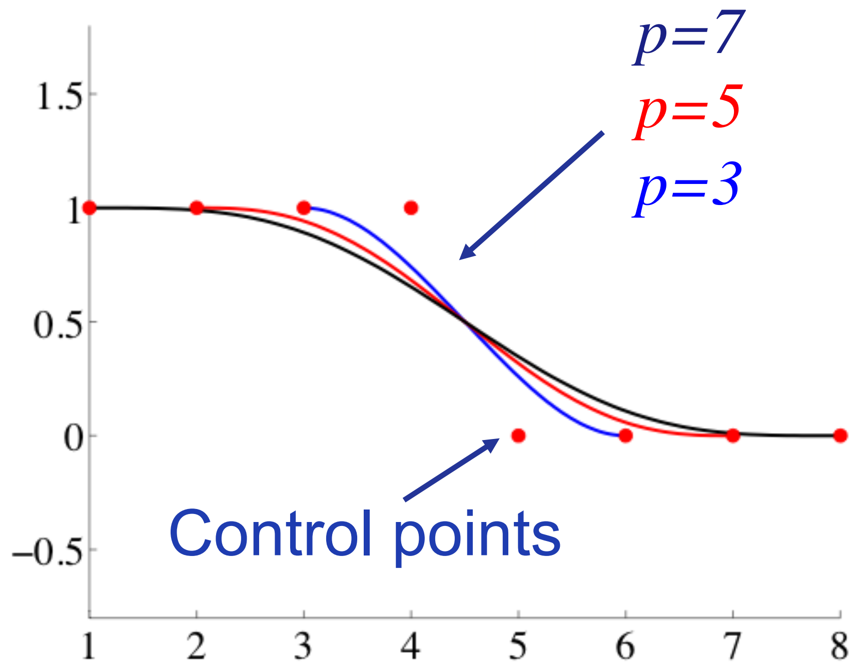


# Variation Diminishing Property

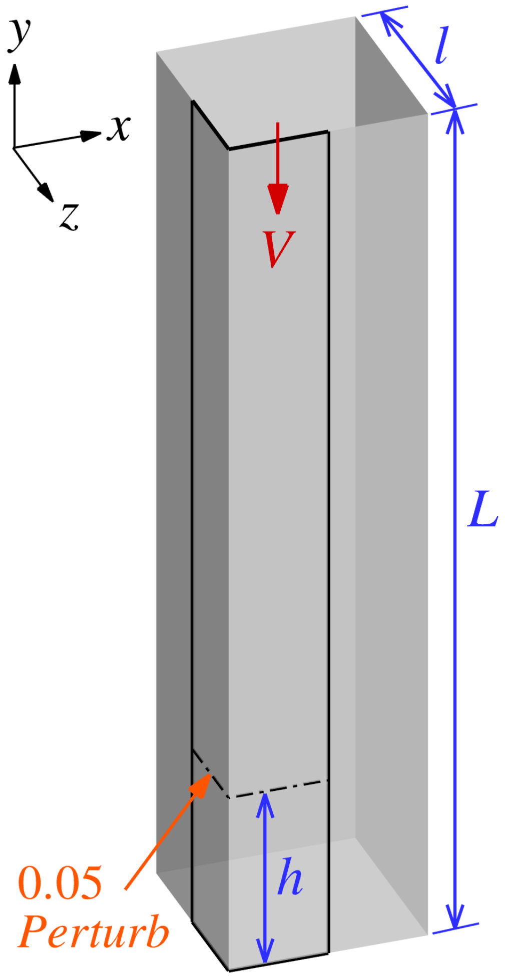
## Lagrange polynomials




## NURBS



# Square Tube Buckling

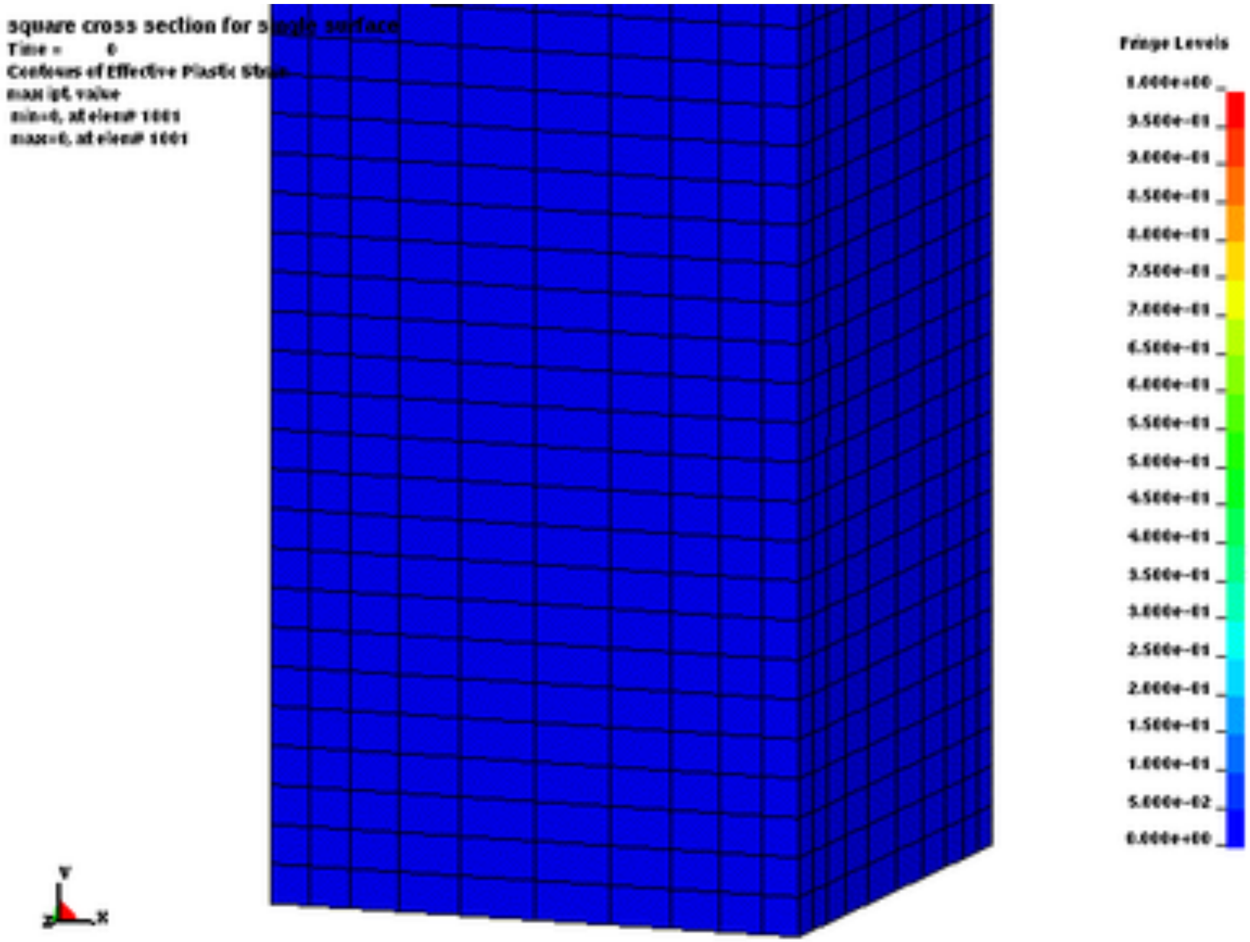



- Standard benchmark for automobile crashworthiness
- Quarter symmetry
- Perturbation to initiate buckling mode
- $J_2$ plasticity with linear isotropic hardening

(LS DYNA, D. Benson *et al.*)

# Smooth Functions are Robust

## $C^3$ quartics in LS DYNA



# IGA and Collocation

1. Use the *strong* variational form of the equations.

2. One quadrature point per node/control point.

3. The ultimate reduced quadrature method.

4. 1D theoretical result*: $O(p\text{-}1)$ in $W^{2,\infty}$ for all $p$ (optimal).

5. Observed numerically in multi-D*:
   $O(p)$ in $L^{\infty}$ and $W^{1,\infty}$ for $p$ even
   $O(p\text{-}1)$ in $L^{\infty}$ and $W^{1,\infty}$ for $p$ odd

---

*F. Auricchio, L. B. Da Veiga, T. J. R. Hughes, A. Reali, and G. Sangalli, "ISOGEOMETRIC COLLOCATION METHODS," *Mathematical Models and Methods in Applied Sciences*, vol. 20, no. 11, pp. 2075–2107, Nov. 2010.
http://www.worldscientific.com/doi/abs/10.1142/S0218202510004878

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---

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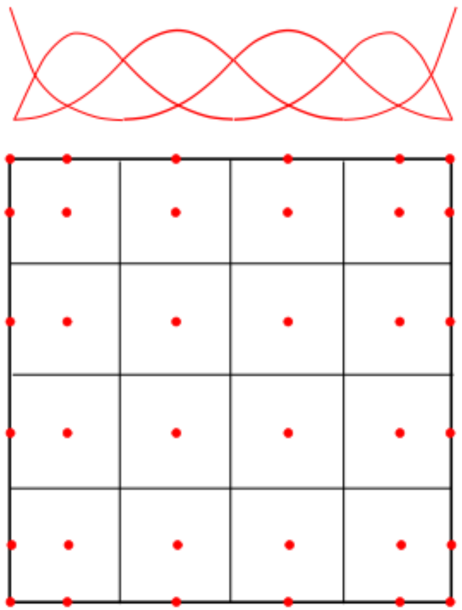
http://www.worldscientific.com/doi/abs/10.1142/S0218202510004878

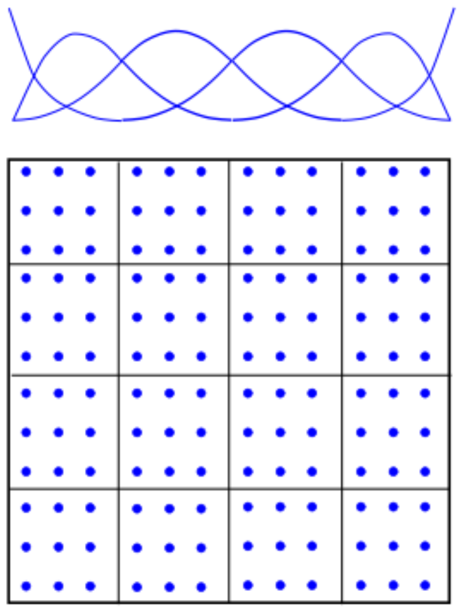
# Quadrature points for *p* = 2

Isogeometric collocation (IGA-C)



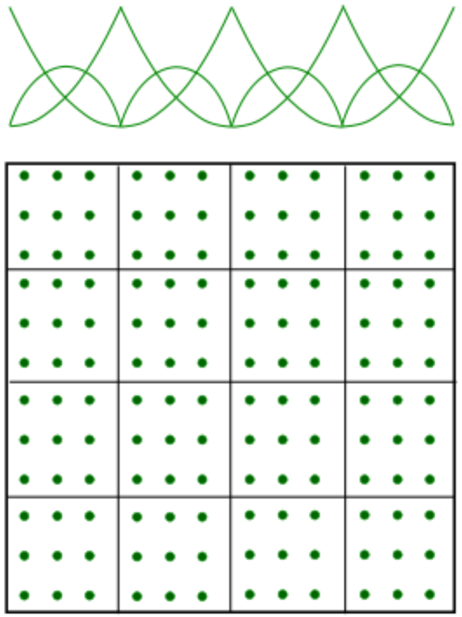
Greville points

Isogeometric Galerkin (IGA-G)



3 X 3 Gauss

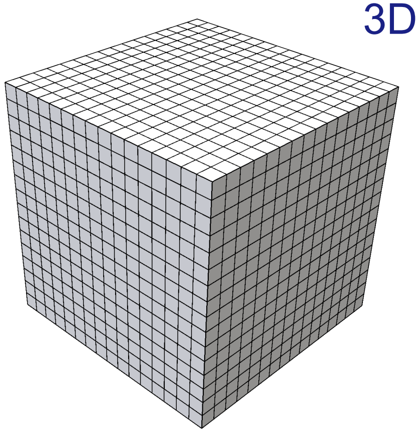
$C^0$ Finite Elements (FEA-G)



3 X 3 Gauss

D. Schillinger, J. A. Evans, A. Reali, M. A. Scott, and T. J. R. Hughes, “Isogeometric collocation: Cost comparison with Galerkin methods and extension to adaptive hierarchical NURBS discretizations,” *Computer Methods in Applied Mechanics and Engineering*, vol. 267, pp. 170–232, Dec. 2013. http://www.sciencedirect.com/science/article/pii/S004578251300193X

# Benchmark problem: Linear elasticity in 3D

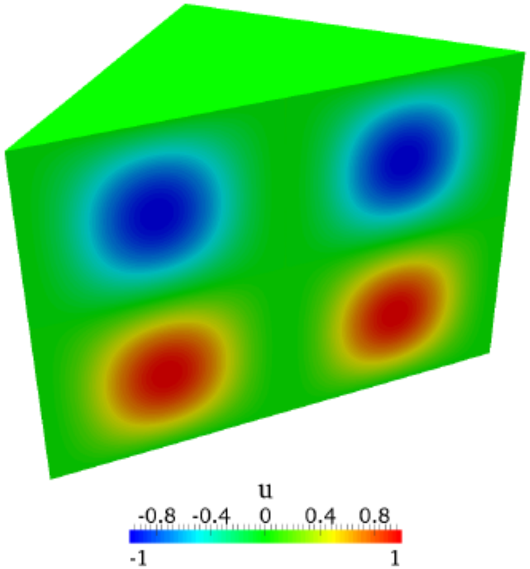


3D domain

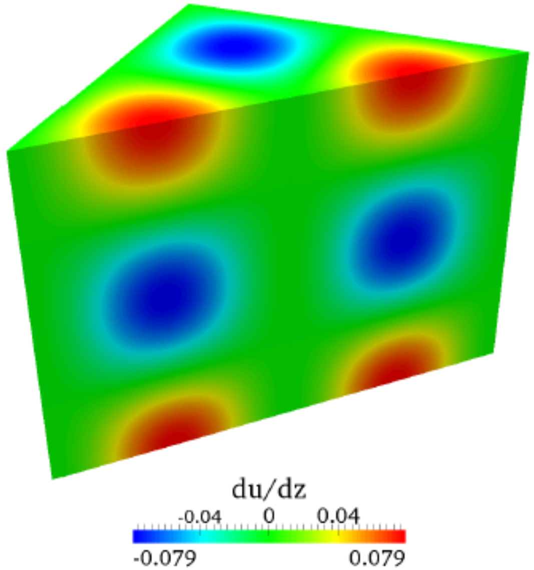
Exact solution:

$$u = v = w = \sin(2\pi x)\ \sin(2\pi y)\ \sin(2\pi z)$$

Displacement field u

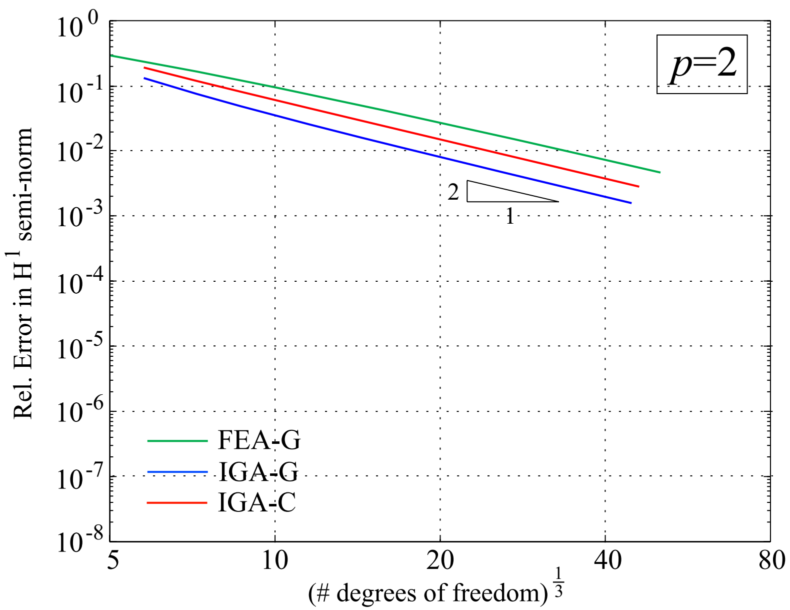


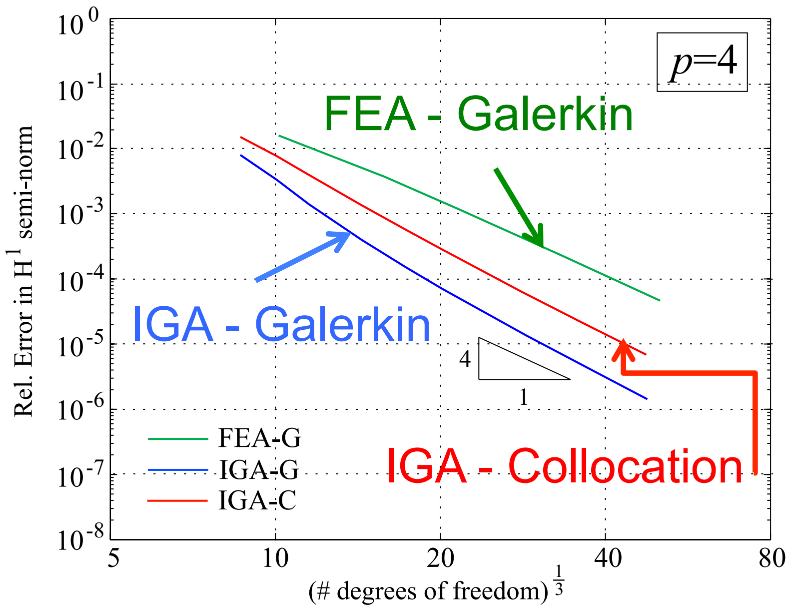

Derivative du/dz




D. Schillinger, J. A. Evans, A. Reali, M. A. Scott, and T. J. R. Hughes, “Isogeometric collocation: Cost comparison with Galerkin methods and extension to adaptive hierarchical NURBS discretizations,” *Computer Methods in Applied Mechanics and Engineering*, vol. 267, pp. 170–232, Dec. 2013. http://www.sciencedirect.com/science/article/pii/S004578251300193X

# Error in $H^1$ semi-norm vs. number of DOF





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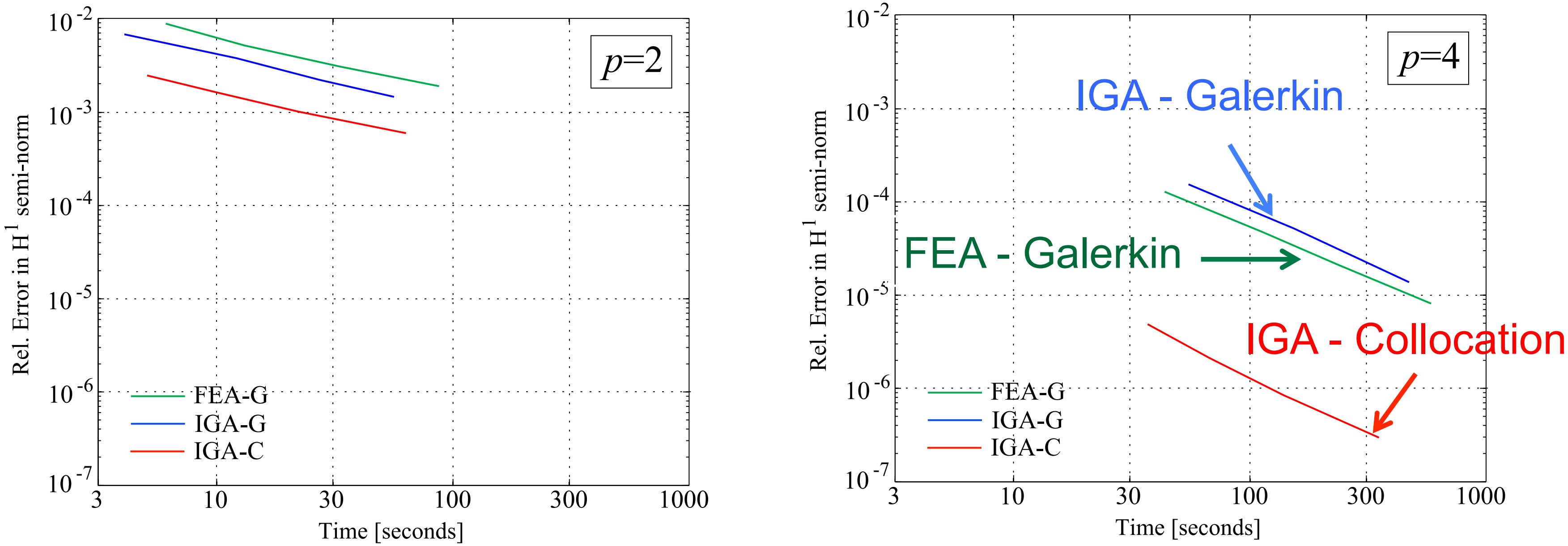
# Error in $H^1$ semi-norm vs. computing time

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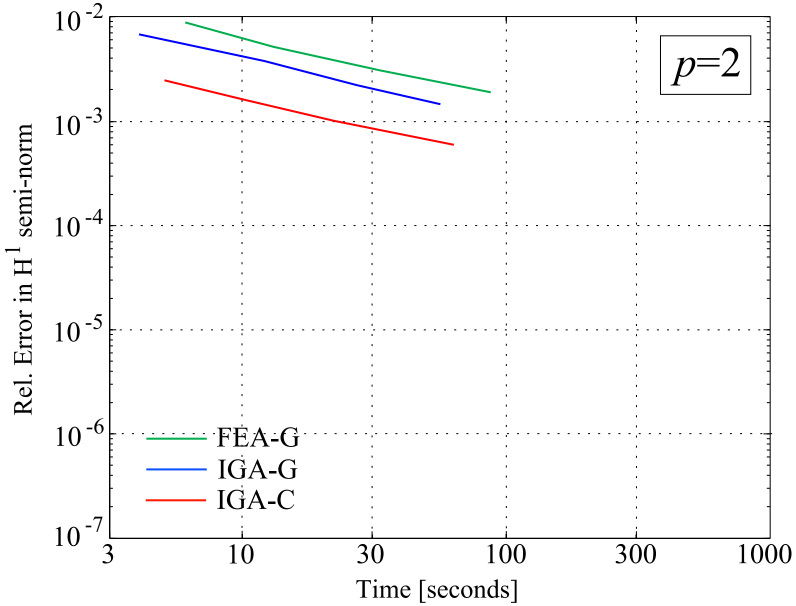


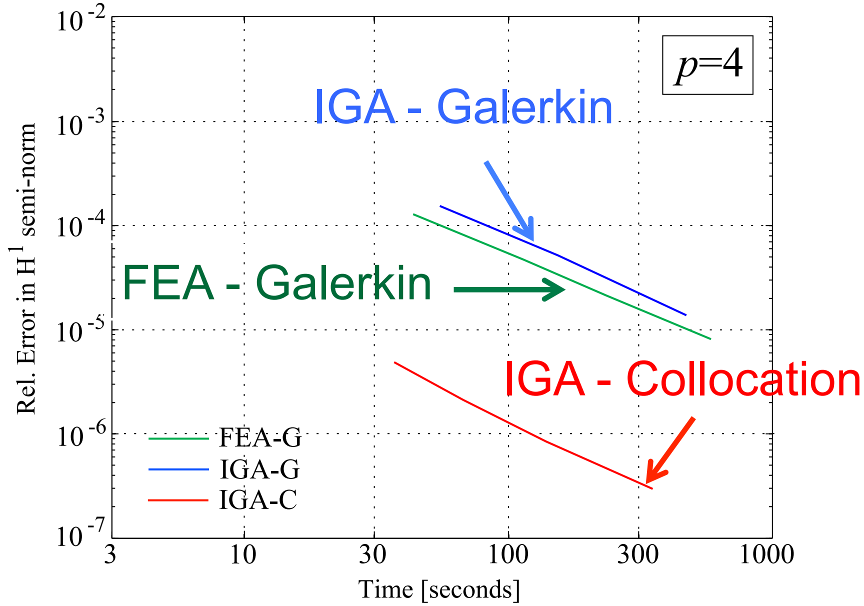


Speed-up: 25 times


D. Schillinger, J. A. Evans, A. Reali, M. A. Scott, and T. J. R. Hughes, "Isogeometric collocation: Cost comparison with Galerkin methods and extension to adaptive hierarchical NURBS discretizations," *Computer Methods in Applied Mechanics and Engineering*, vol. 267, pp. 170–232, Dec. 2013. http://www.sciencedirect.com/science/article/pii/S004578251300193X

# Breakthrough in IGA Collocation

- “The Variational Collocation Method,” H. Gomez, L. De Lorenzis, *CMAME*, accepted, 2016.

# Breakthrough in IGA Collocation

- "The Variational Collocation Method," H. Gomez, L. De Lorenzis, *CMAME*, accepted, 2016.
- There exist collocation points, so-called *Cauchy-Galerkin points,* that produce the Galerkin solution exactly, for all $p$, odd as well as even.

# Breakthrough in IGA Quadrature

- "Fast Formation of Isogeometric Galerkin Matrices by Weighted Quadrature," F. Calabrò, G. Sangalli, and M. Tani, *CMAME*, accepted, 2016.

- http://arxiv.org/abs/1605.01238v1

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- http://arxiv.org/abs/1605.01238v1
- Much greater efficiency for Galerkin matrices than classical element-by-element implementation.

# Breakthrough in IGA Quadrature

- Example:
  - Formation and assembly of a Galerkin mass matrix.

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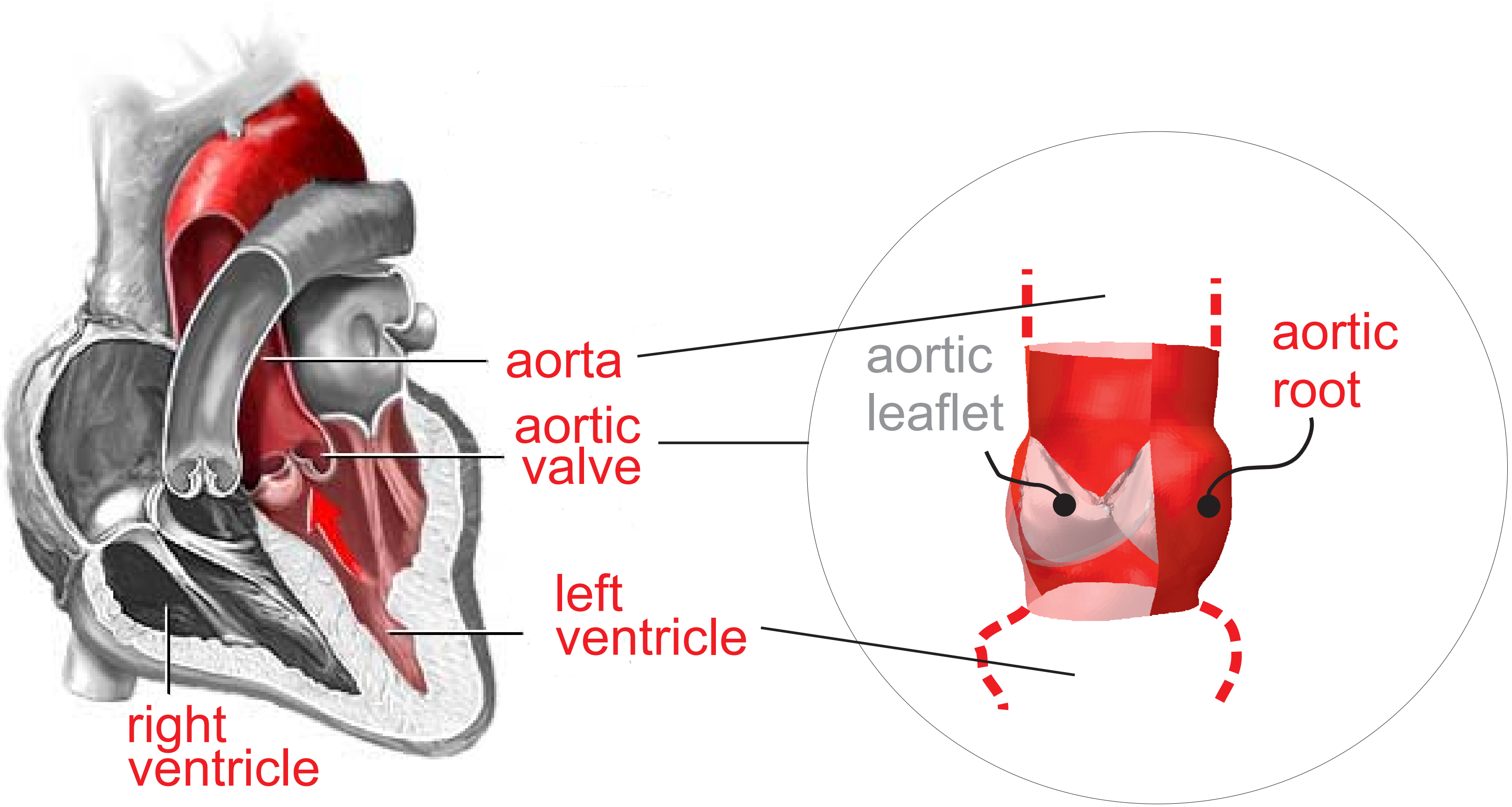
  - Time = 62 hours.
  - New procedure = 27 seconds!
  - Speedup factor > 8,000!

# Applications

# Aortic Valve




“Patient-specific isogeometric structural analysis of aortic valve closure,” S. Morganti, F. Auricchio, D. Benson, F.I. Gambarin, S. Hartmann, TJRH, A. Reali, *CMAME*, 2015.

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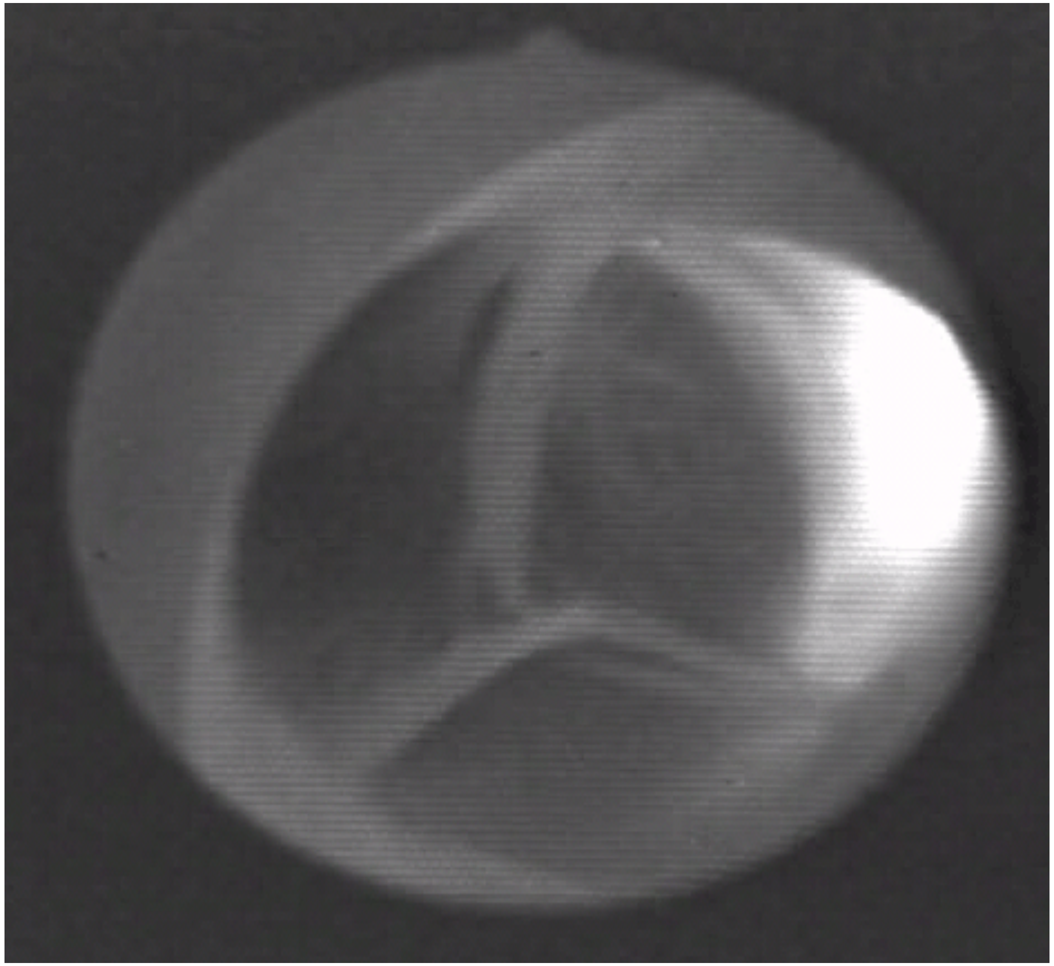
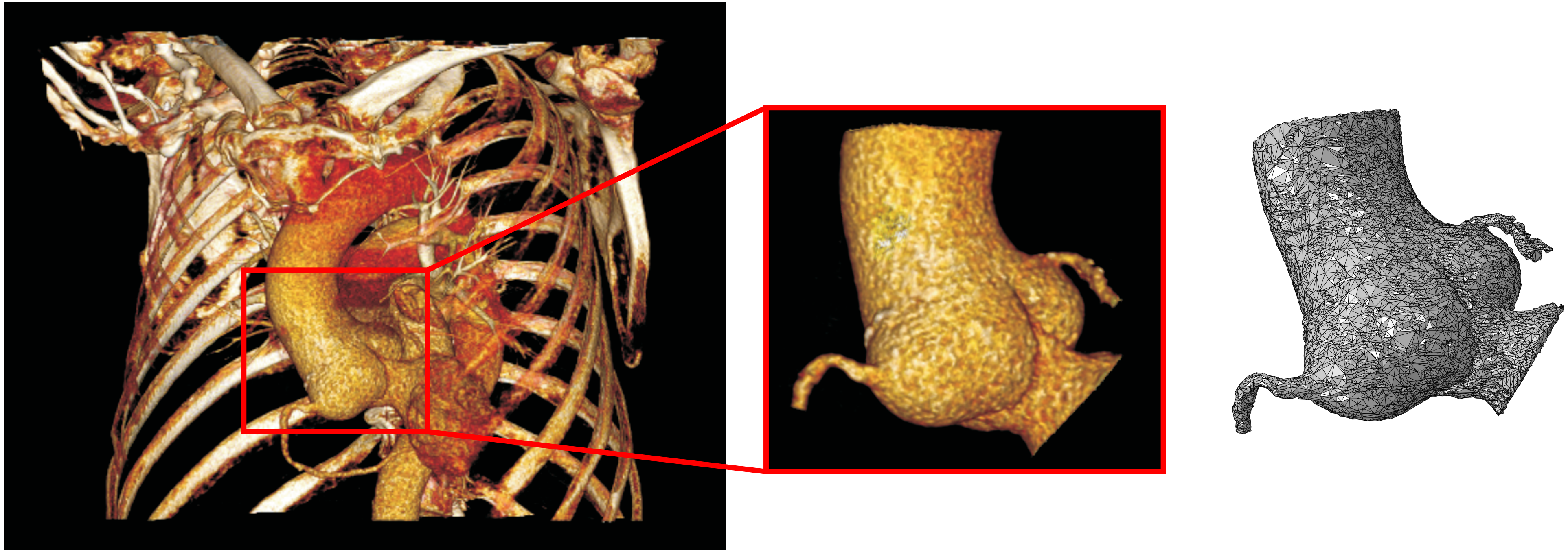


# CTA to STL file



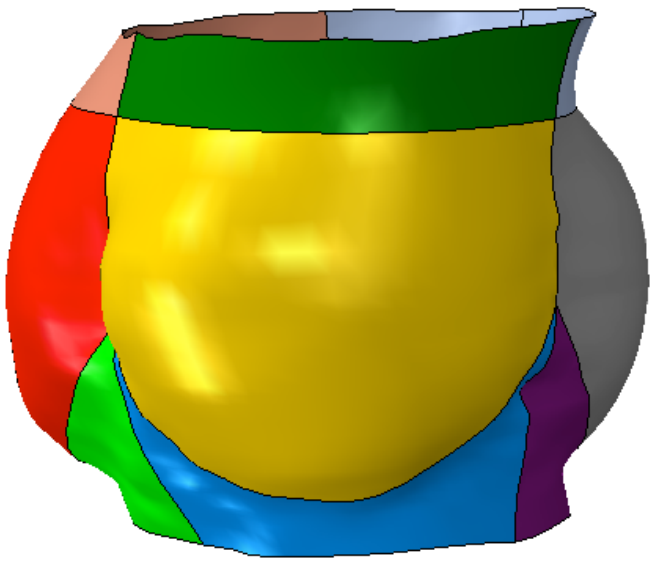
(a) (b) (c)

(a) Primary 3D reconstruction obtained using OsiriX

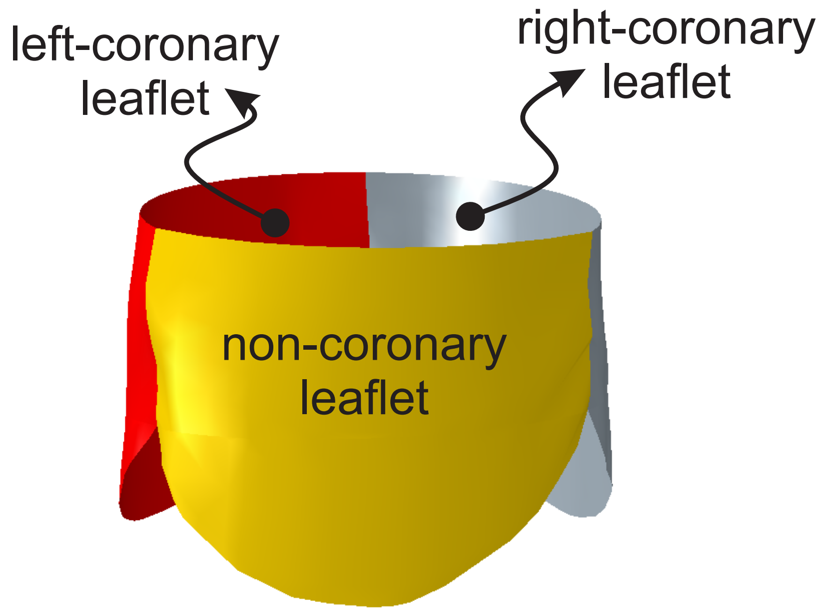
(b) 3D specific reconstruction of the aortic root after cropping and segmentation

(c) STL representation of the extracted region of interest.

# Multi-patch aortic valve geometry

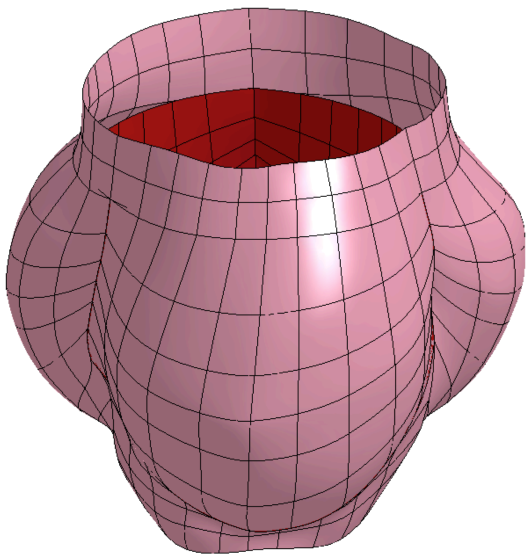


Aortic root subdivided into nine NURBS patches

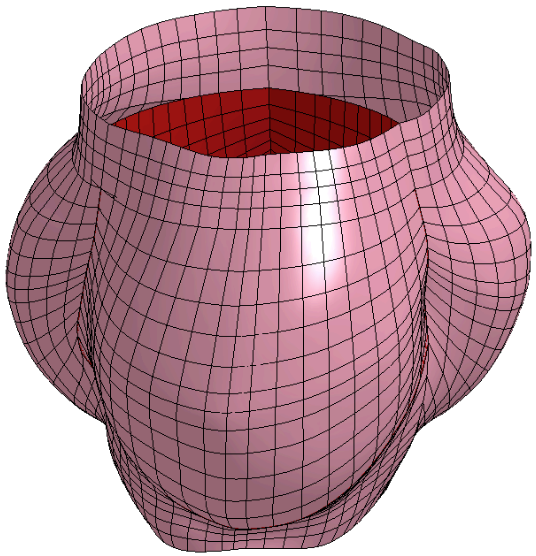



Each leaflet represented by a single NURBS patch

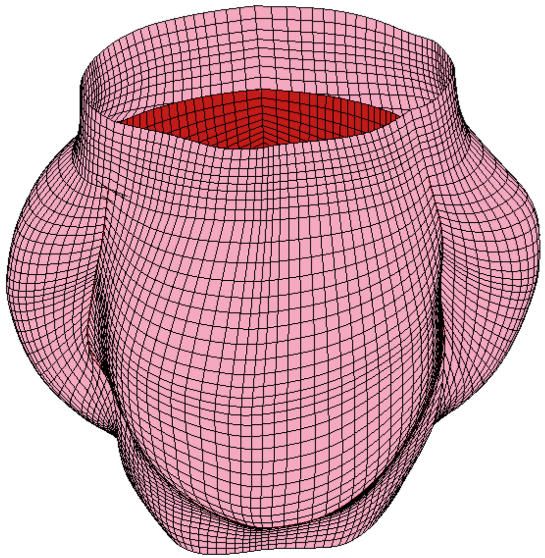
# NURBS meshes for patient-specific aortic root and leaflets



Coarse mesh
(762 control points)



Medium mesh
(2890 control points)

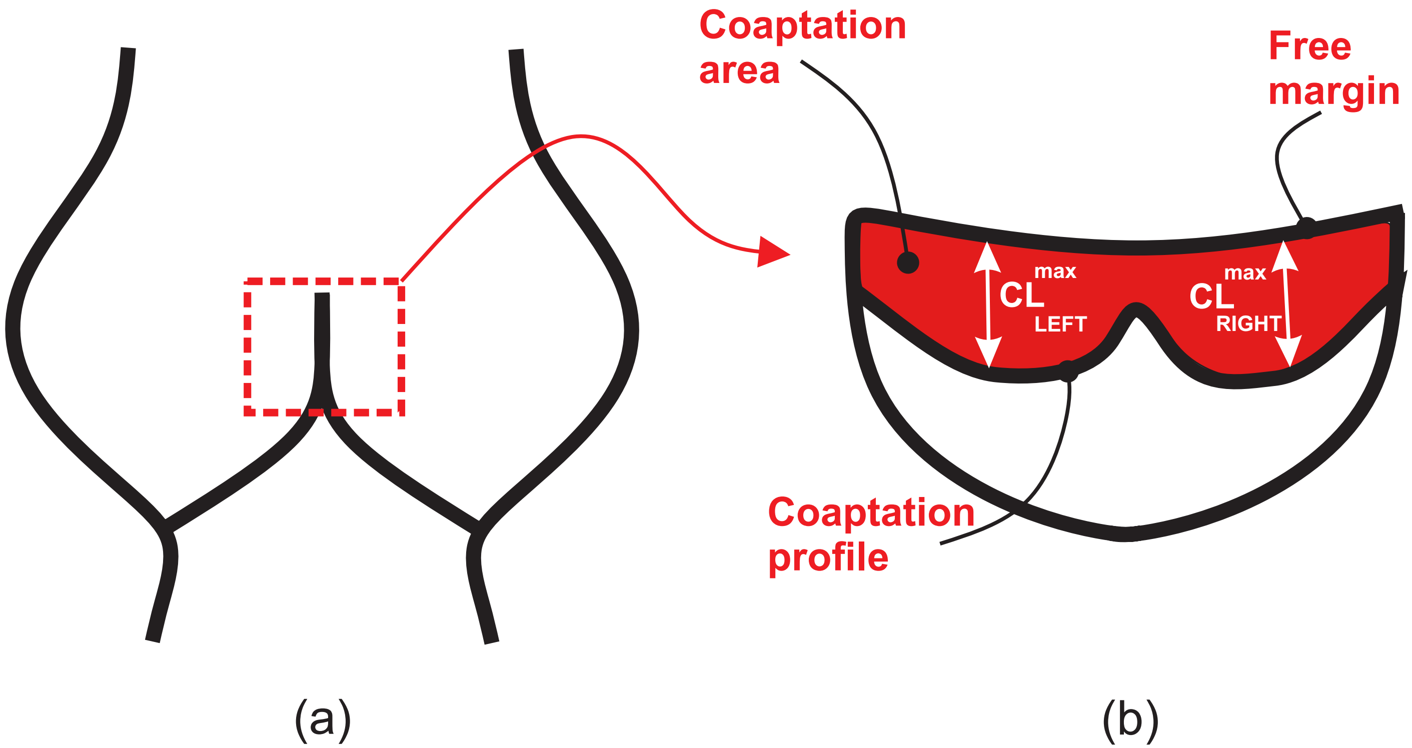


Fine mesh
(9396 control points).

1. Reissner-Mindlin shell theory for the aortic root.

2. Kirchhoff-Love rotation-free shell theory for the aortic valve leaflets.

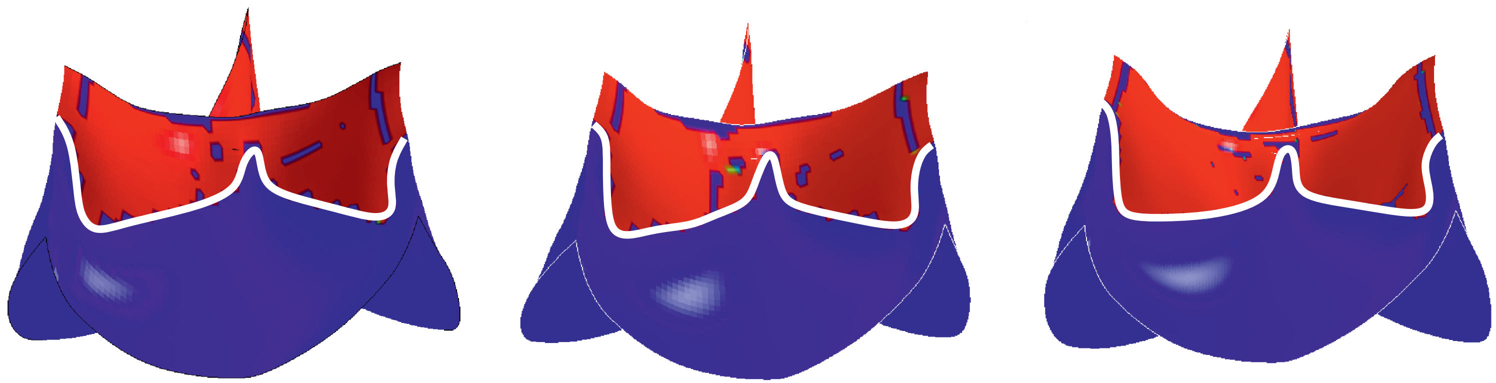
# Coaptation Profile




(a) Longitudinal section of the aortic valve during diastole

(b) Coaptation area, the leaflet free margin, and coaptation profile for one leaflet

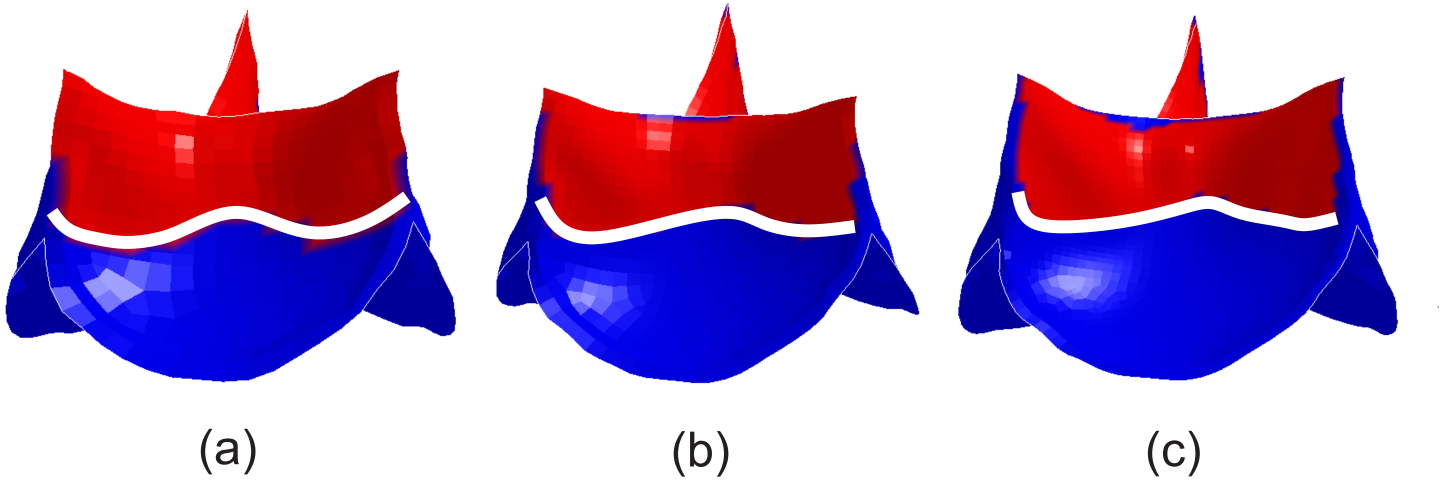
# IGA: Coaptation Profile with LS-DYNA



(a) (b) (c)

(a) 762 nodes (b) 2890 nodes (c) 9396 nodes

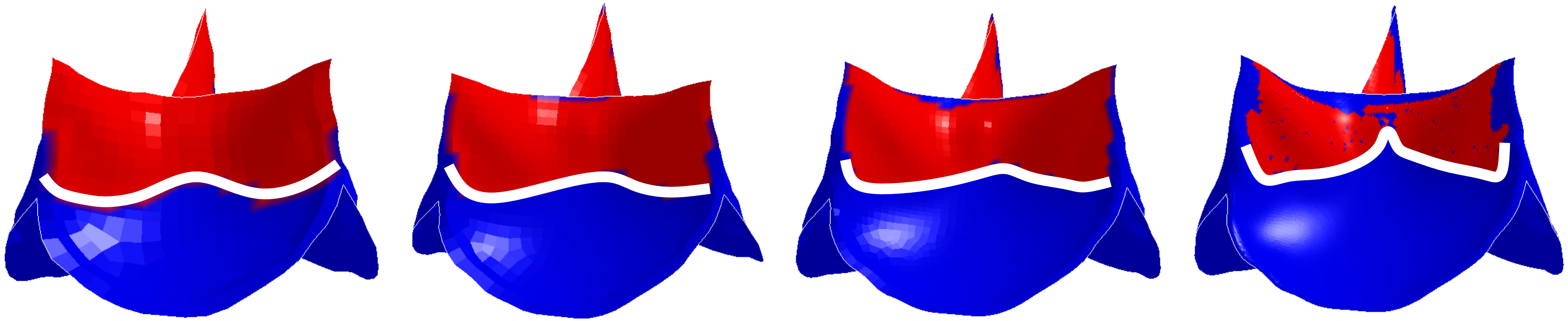
# FEA*: Coaptation Profile with LS-DYNA



(a) 6446 nodes (b) 14329 nodes (c) 37972 nodes

*Belytschko-Tsay four-node Reissner-Mindlin shell finite elements

# FEA*: Coaptation Profile with LS-DYNA



(a) (b) (c) (d)

(a) 6446 nodes (b) 14329 nodes (c) 37972 nodes (d) 153646 nodes

*Belytschko-Tsay four-node Reissner-Mindlin shell finite elements

# Coaptation length for IGA and FEA

| Analysis | # nodes | # DOF | Coaptation Length | |
|---|---|---|---|---|
| | | | $CL_{max}^{(left)}$ [mm] | $CL_{max}^{(right)}$ [mm] |
| IGA | 762 | 3708 | 9.30 | 9.40 |
| | 2890 | 19476 | 9.25 | 9.40 |
| | 9396 | 50496 | 9.30 | 9.35 |
| FEA | 1112 | 6672 | 11.1 | 12.8 |
| | 3117 | 18702 | 10.8 | 10.2 |
| | 6446 | 38676 | 10.4 | 9.80 |
| | 14329 | 85974 | 9.70 | 9.70 |
| | 37972 | 227832 | 9.45 | 9.50 |
| | 153646 | 921876 | 9.30 | 9.35 |

# Solution times for comparable accuracy

| Analysis | # Nodes | # CPUs | Time step | # Increments | Total analysis time |
|---|---|---|---|---|---|
| IGA | 762 | 12 | 2.30e-07 | 4347390 | 1h 15m |
| FEA | 153646 | 12 | 2.65e-08 | 37787314 | 550h 23m |

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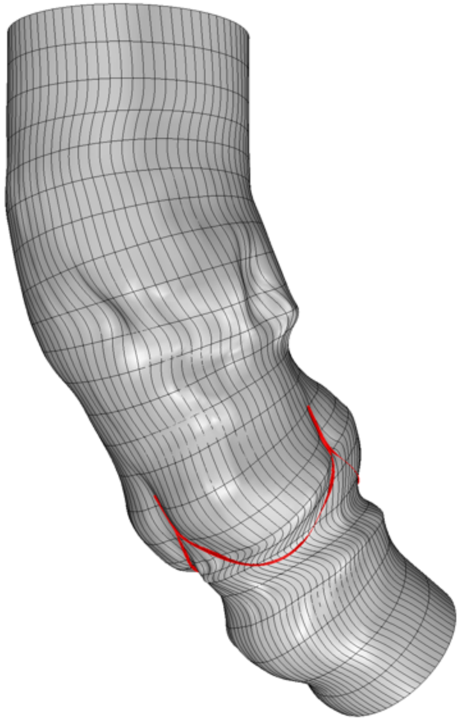
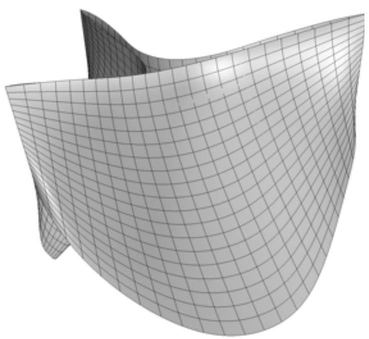
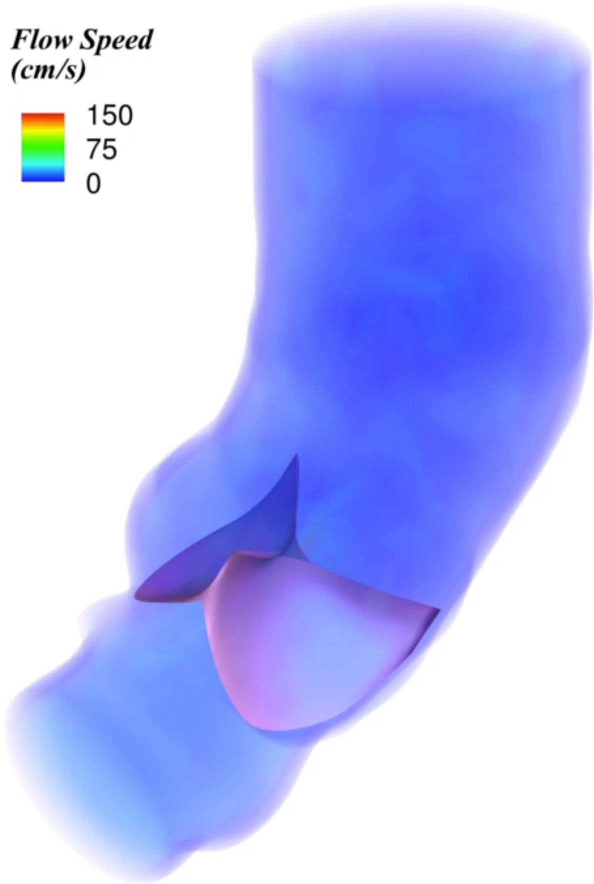
Why is IGA so much faster than traditional FEA?

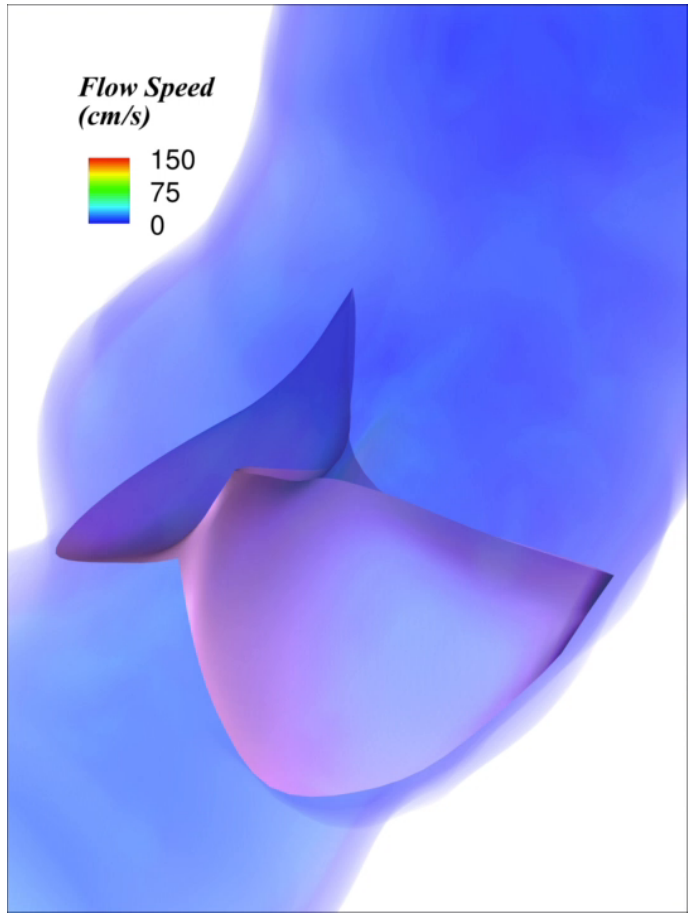
1. Much more accurate per degree of freedom.
2. Efficient dynamics, e.g., large time steps.
3. Quality of contact surface provided by smooth geometry and smooth basis functions.

# ALE / Immersed Kirchhoff-Love Shell





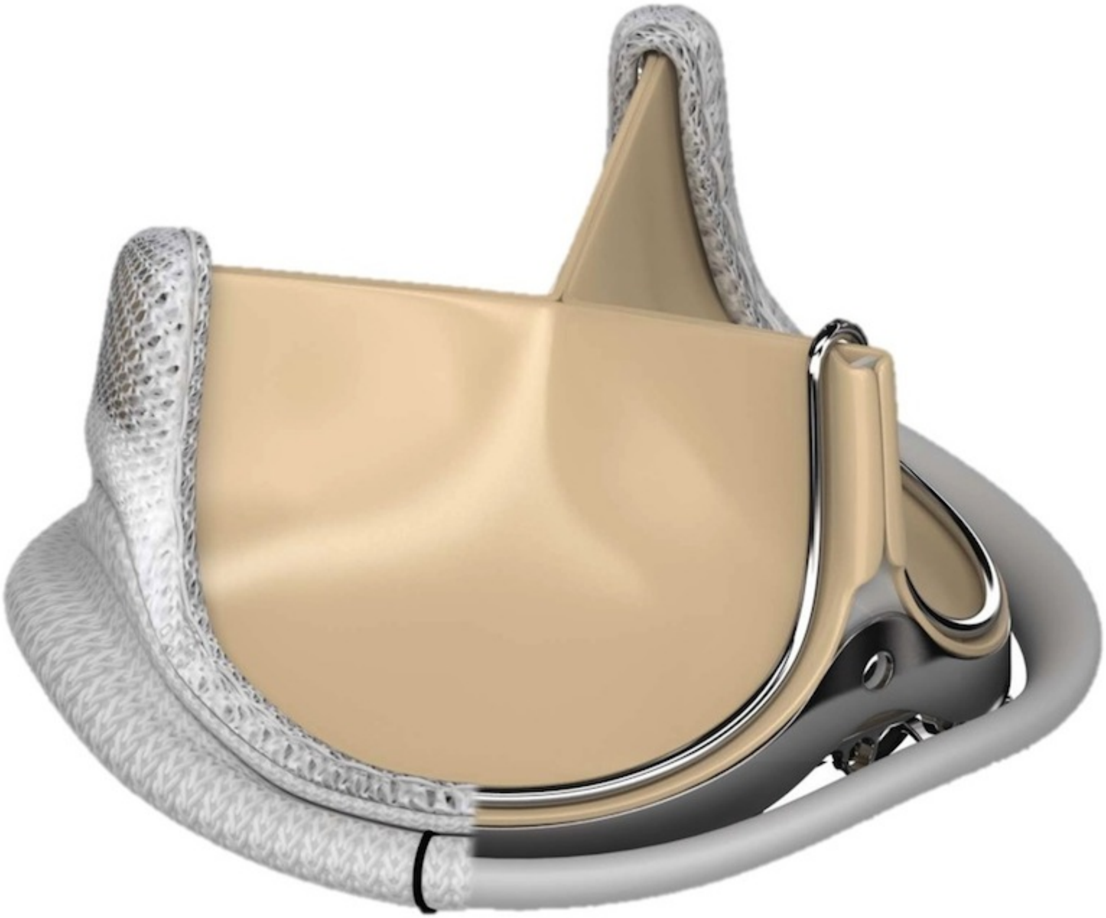


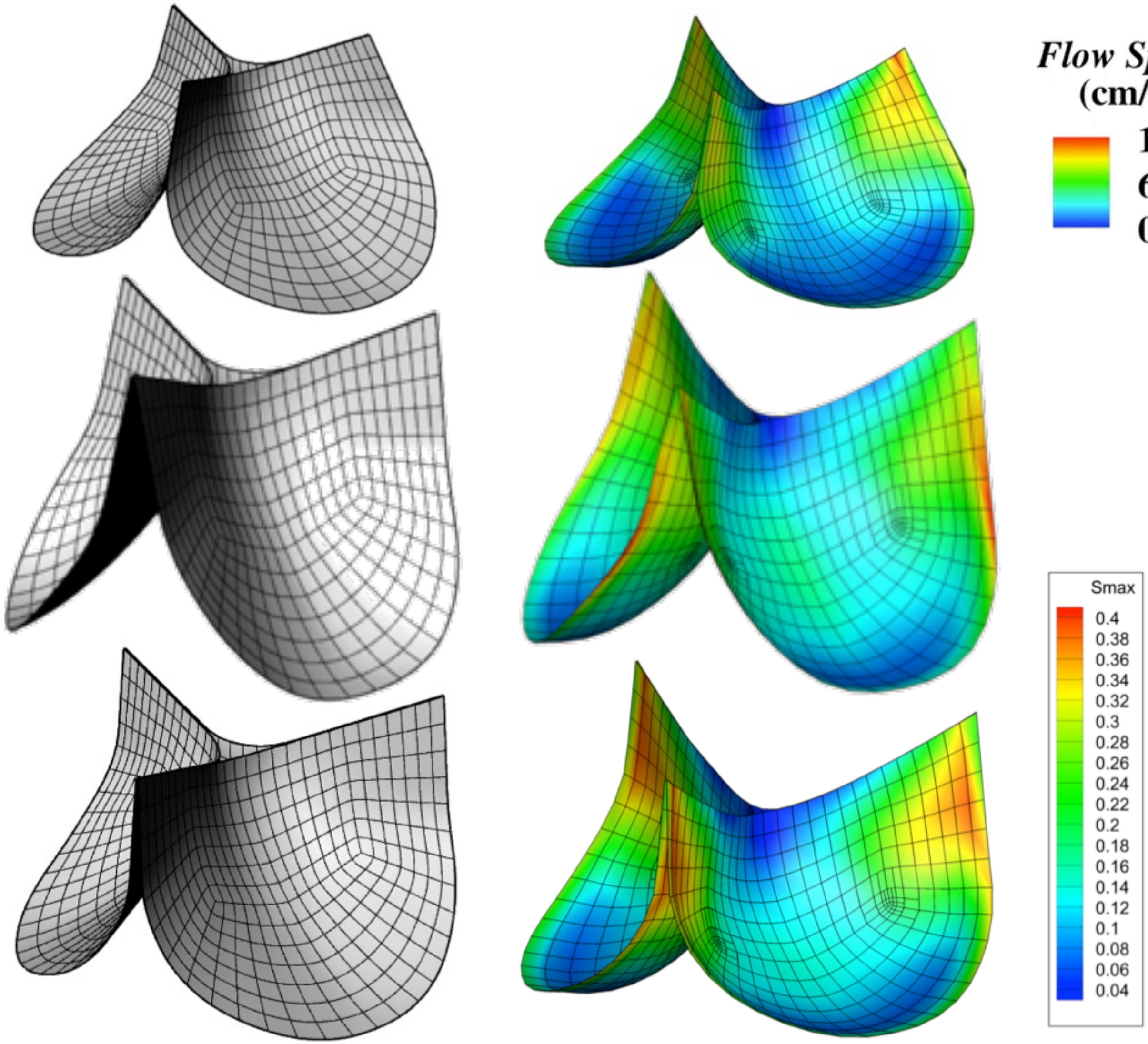

Patient-specific volumetric NURBS artery wall

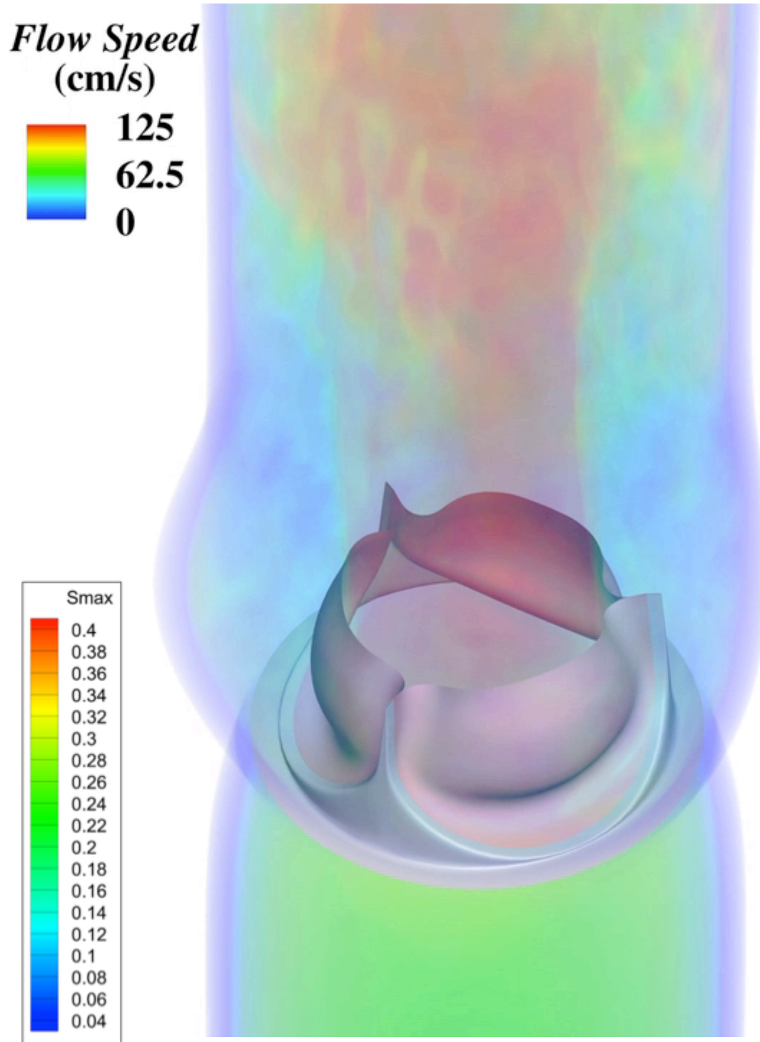
M.-C. Hsu, S. Morganti, A. Reali, F. Auricchio, J. Kiendl, D. Kamensky, M. Sacks, et al. 2016

# Bioprosthetic Heart Valve



# ALE / Immersed Kirchhoff-Love Shell




Static closing analysis of different designs

M.-C. Hsu, A. Herrema, et al., 2015

Volumetiric NURBS artery wall

+ M. Sacks, D. Kamensky, et al., 2015

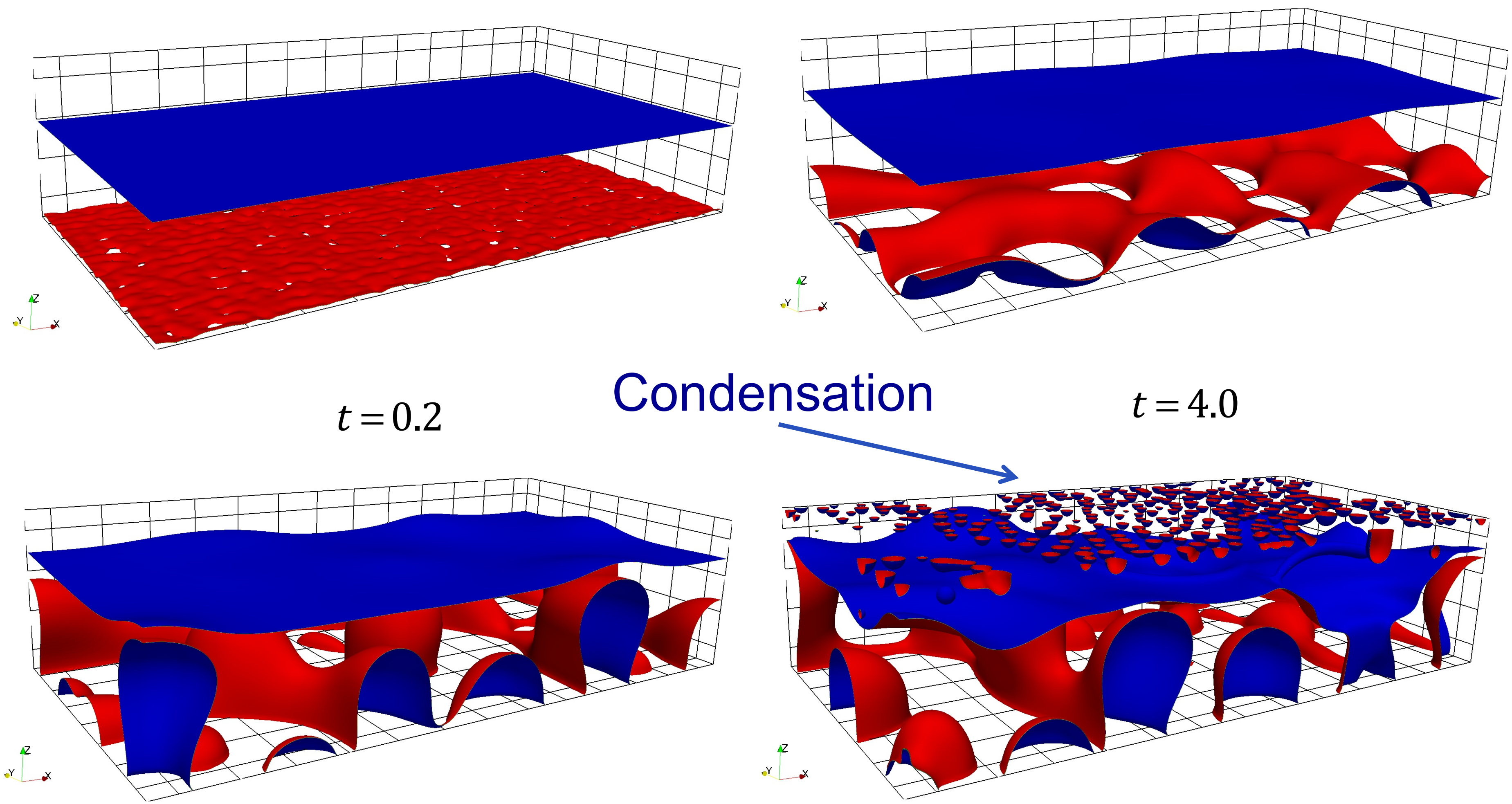
# Boiling

- NOVA, a science TV show:
  - Does mathematics explain the physical world?
- One man's opinion:
  - "No!  One of the things it cannot simulate is boiling"

Ju Liu does not agree

- Navier-Stokes-Korteweg equations – $3^{rd}$ derivatives

# Three-dimensional Boiling (J. Liu et al.)



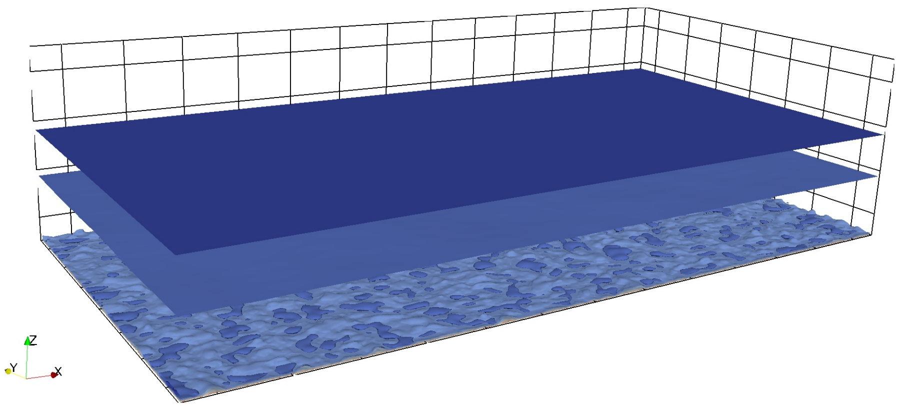

$t = 0.2$

$t = 4.0$

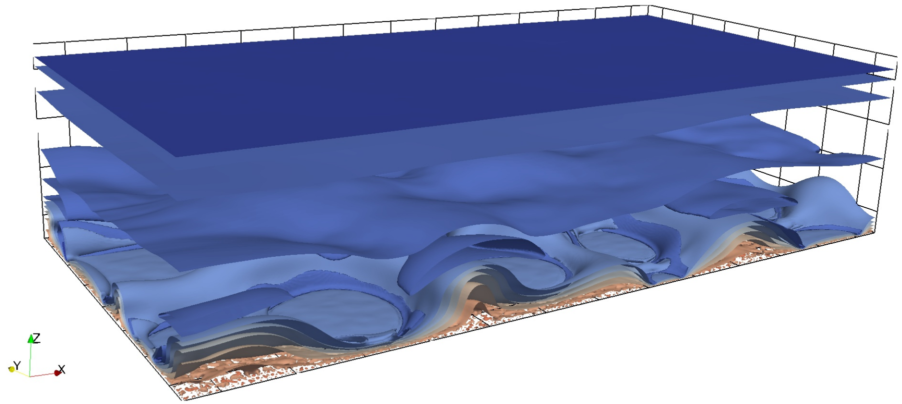
$t = 8.0$

$t = 12.0$

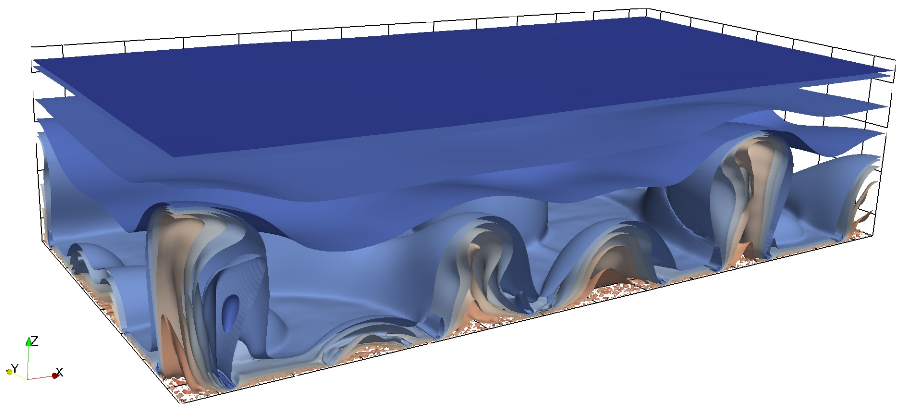
# Three-dimensional Boiling (J. Liu et al.)



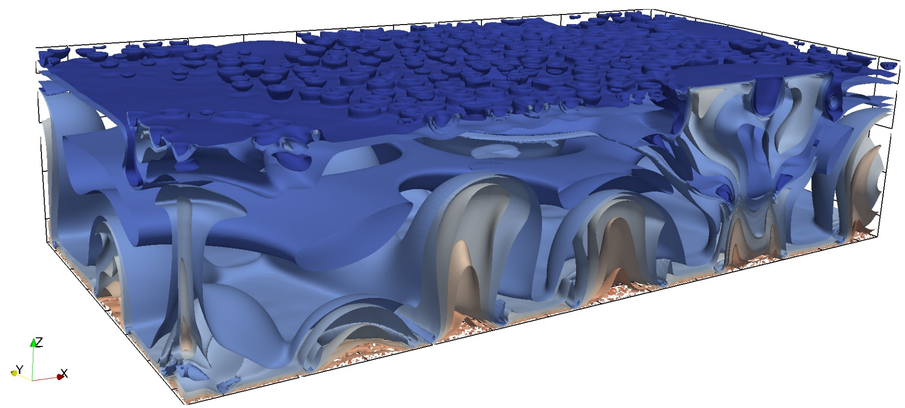
$t = 0.2$



$t = 4.0$



$t = 8.0$



$t = 12.0$

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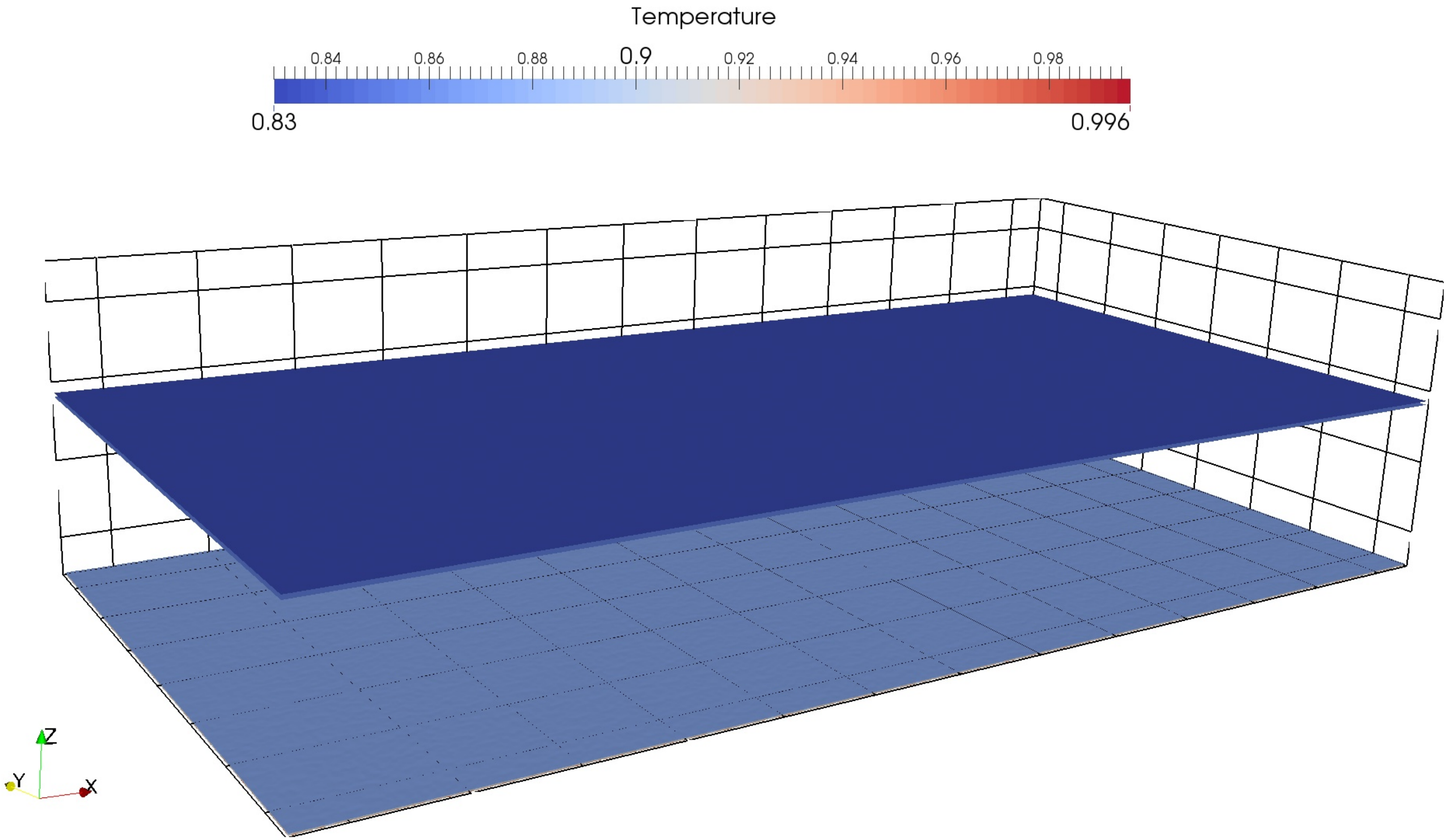
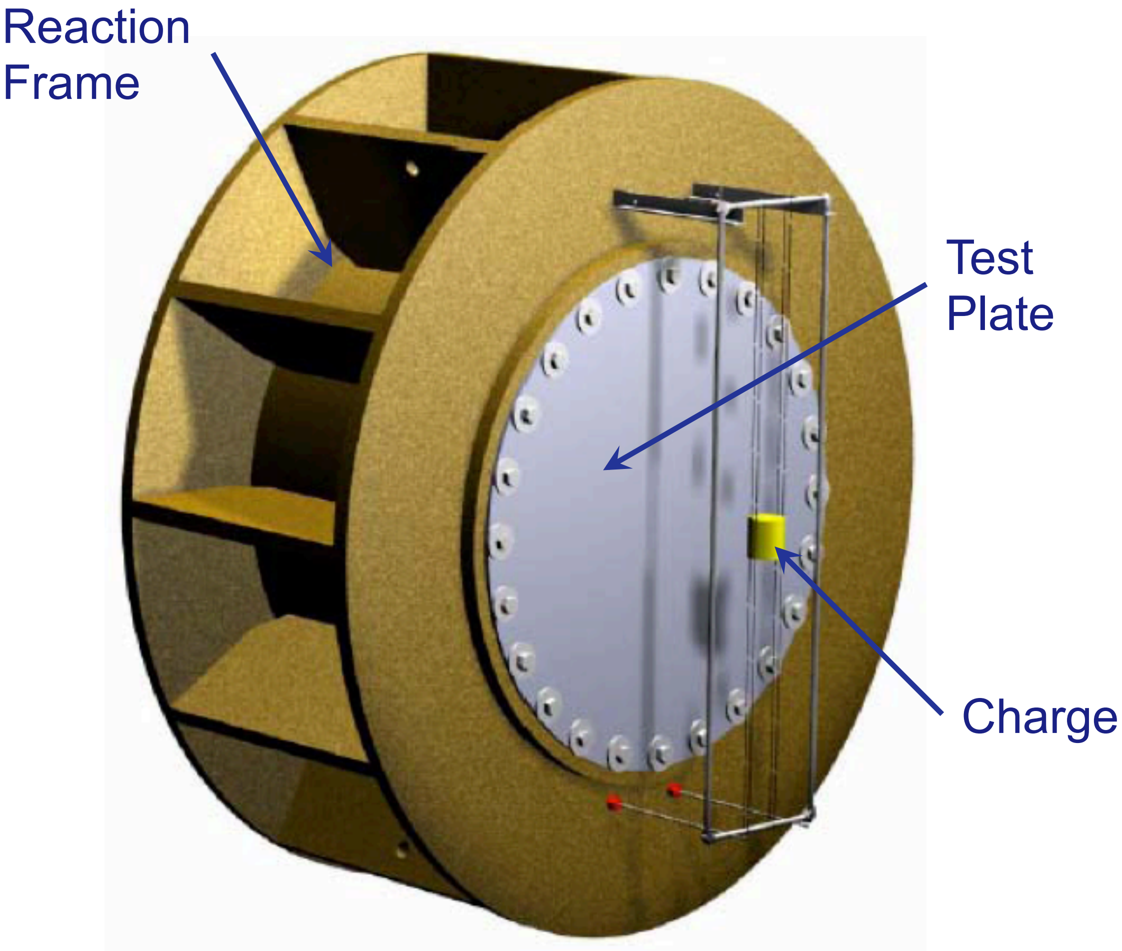


# Ductile Fracture

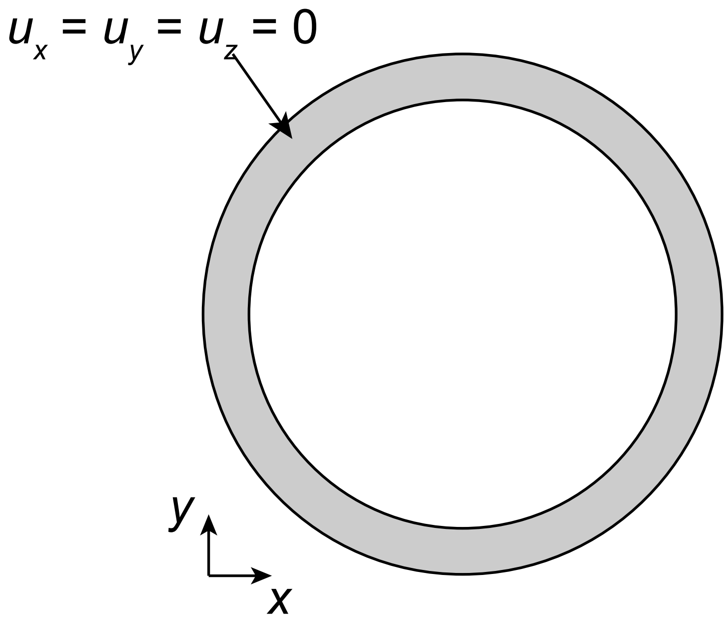
# Circular Plate Subject to Impulse Load

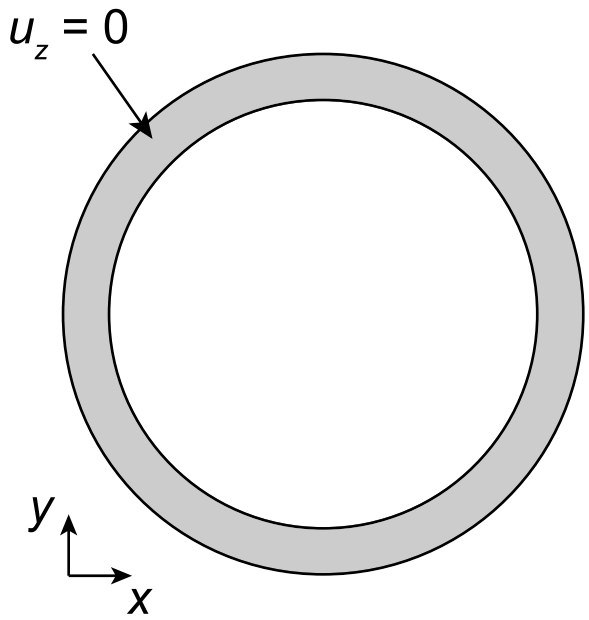





Figures from K.G. Webster, *Investigation of Close Proximity Underwater Explosion Effects on a Ship-Like Structure Using the Multi-Material Arbitrary Lagrangian Eulerian Finite Element Method*, Master's Thesis, Virginia Polytechnic Institute and State University, 2007.

# Displacement Boundary Conditions



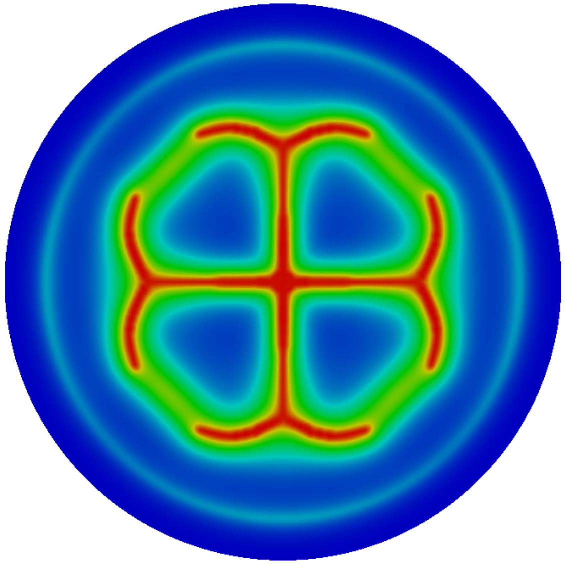




**Clamped BC:** No displacement in any direction on outer ring

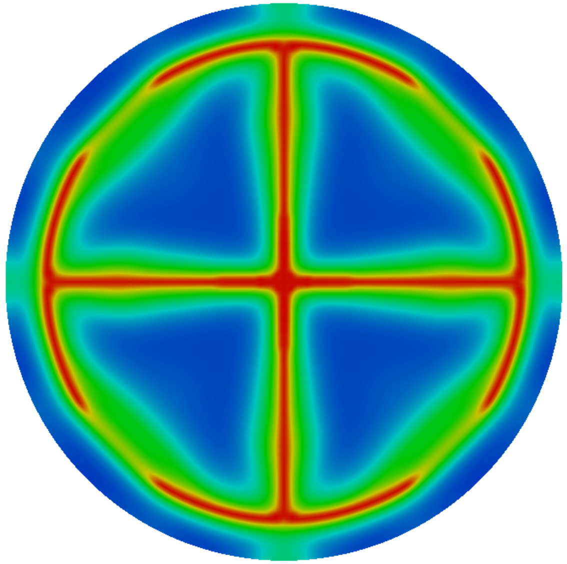
**Sliding BC:** No displacement on outer ring in *z*-direction

# Comparison of BCs

Clamped



Sliding



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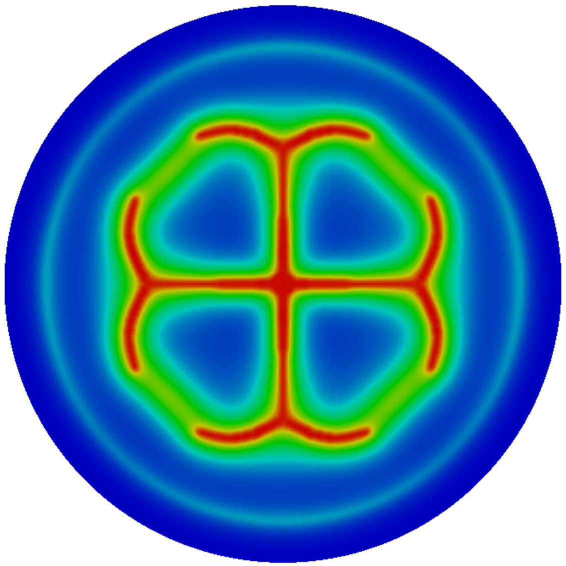
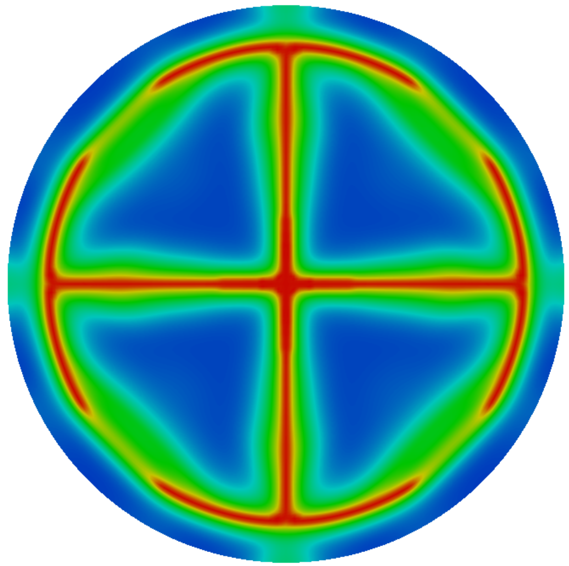
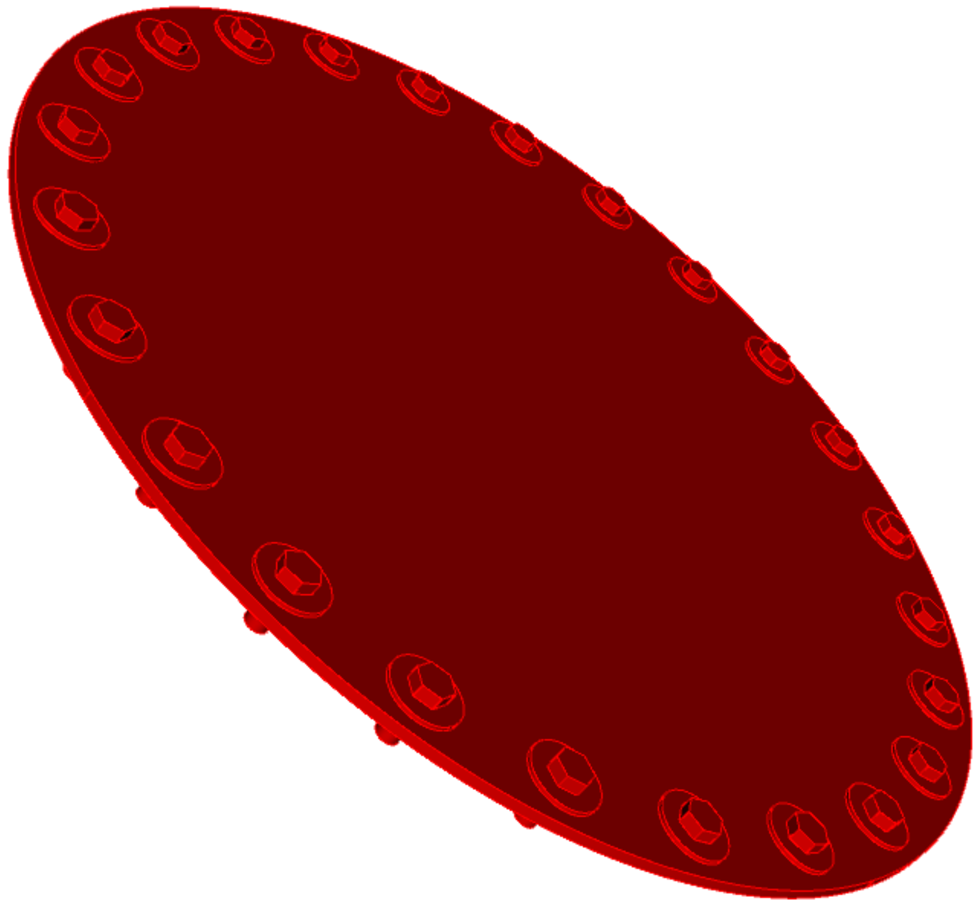


Sliding



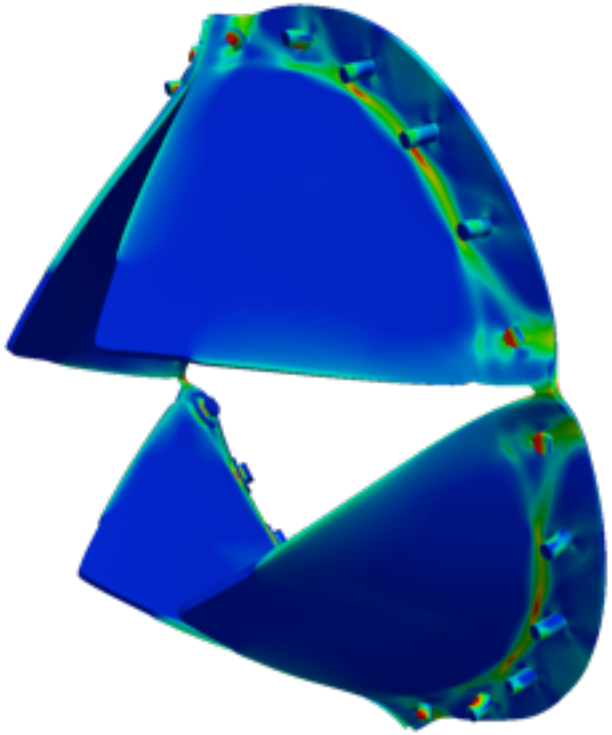
*"Everything should be made as simple as possible, but not simpler."* A. Einstein (?)

# NURBS Circular Plate Model*

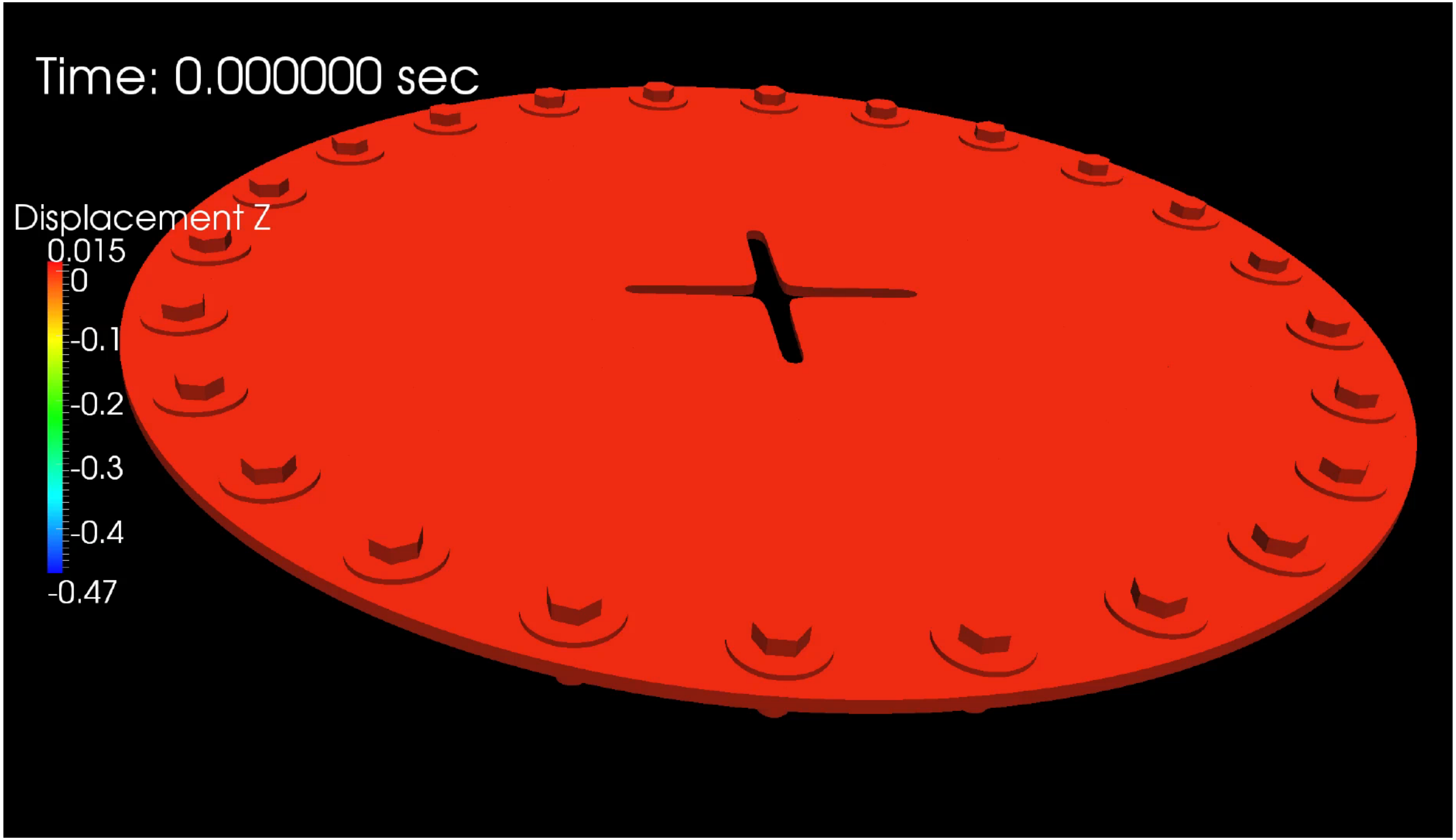


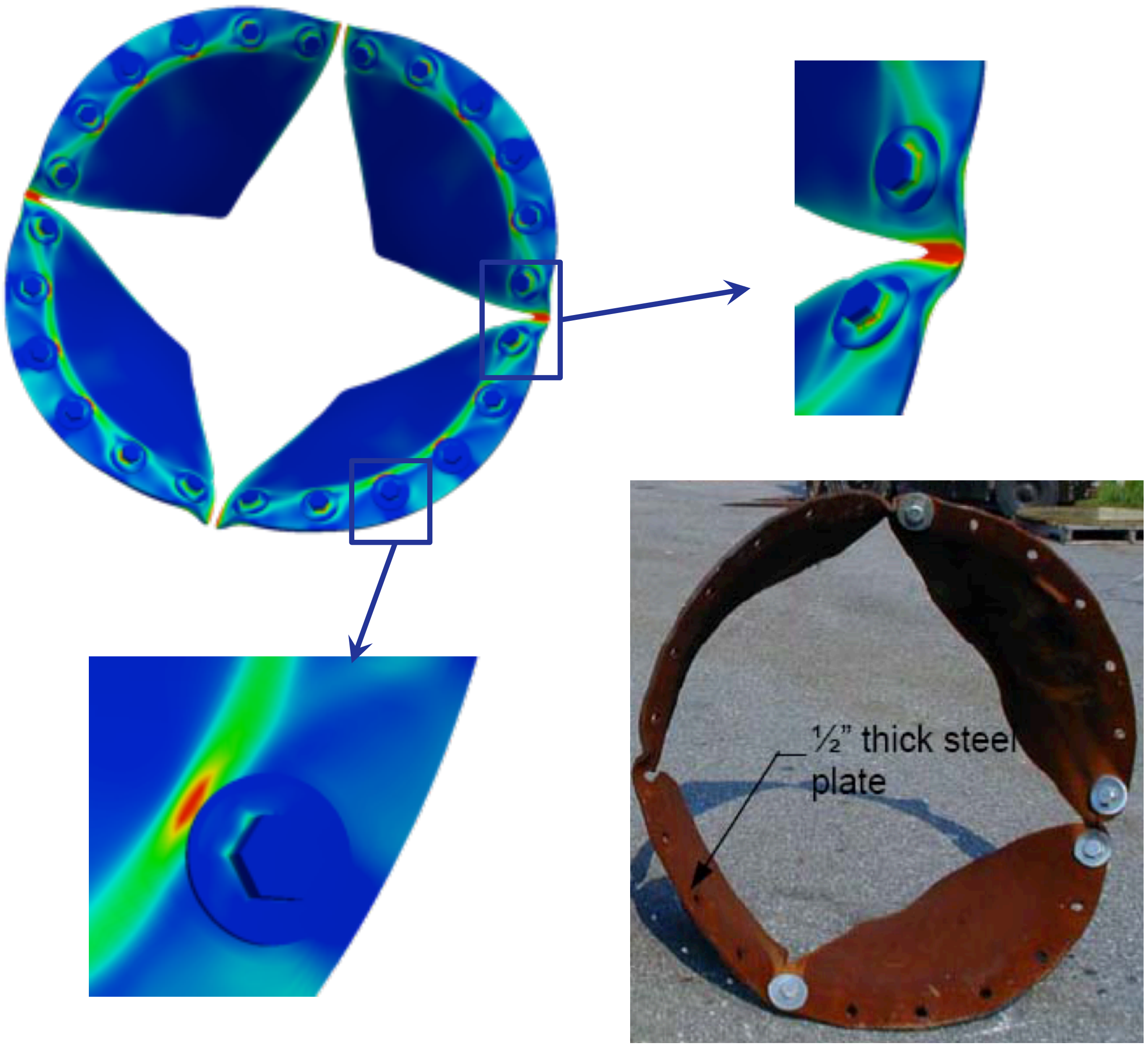
Includes bolts and washers

* M.J. Borden, T.J.R. Hughes, C. Landis, A. Anvari, I. Lee, 2016




½" steel plate


Time: 0.000000 sec
Displacement Z
0.015
0
-0.1
-0.2
-0.3
-0.4
-0.47


½" thick steel
plate

# Isogeometric Analysis: Summary

- One of the most active areas of FEA and CAGD research
- Overarching goal: Improve engineering product design
- Focus so far: The design-through-analysis process
- "Better, faster, cheaper"
  - Improve quality of analysis
  - Expedite analysis model development
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- A fruitful, promising and growing area of research
- Gaining traction in industry

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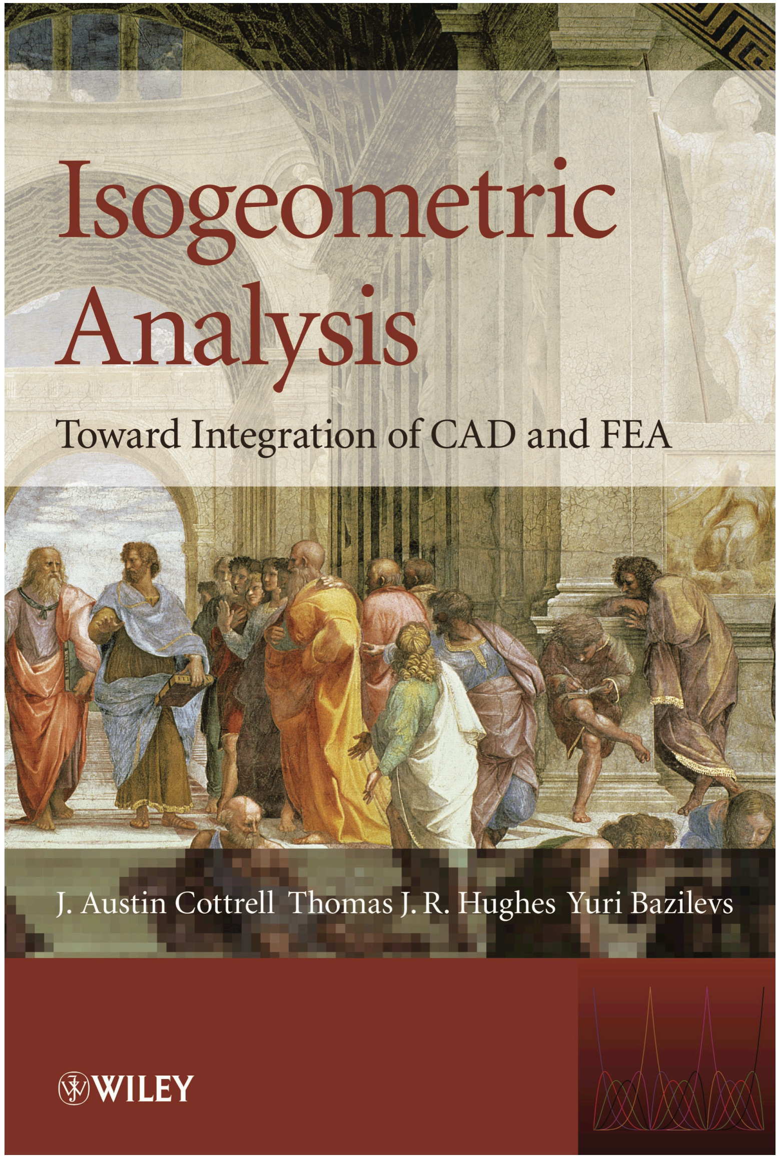
1) It can't be done.

2) It probably can be done, but it's not worth doing.

3) I knew it was a good idea all along!




Published in 2009