MODELING FLOW IN LARGE VUG CRETACEOUS CARBONATES

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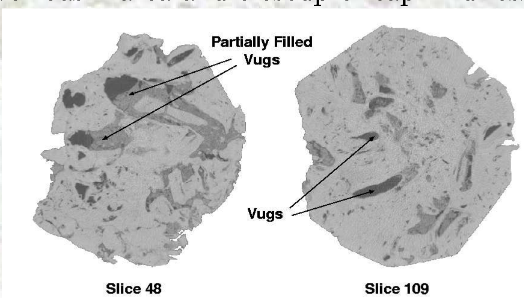
A *vug* is a cavity much larger than the pore size. Vugs are common in carbonate rocks.

Pipe Creek Reef Outcrop in Central Texas

River basin area and closeup of caprinid fossils.



CT Scans of $30 \times 30 \times 36$ cm^3 sample

Remark: The cm-scale vugs are connected by narrow channels of aperture about 2 mm.

Overview of Project

Objective: Predict the average macro-scale flow.

Methodology: Use mathematical homogenization on the micro-scale to obtain the macro-scale flow.

The Micro-Scale Governing Equations

Vuggy region: Ω_s

$$-2\mu\nabla\cdot D\mathbf{u} + \nabla p = \mathbf{f} \quad \text{Stokes Equations}$$
$$\nabla\cdot\mathbf{u} = q \quad \text{Conservation}$$

Rock matrix: Ω_d

$$\mu K^{-1}\mathbf{u} + \nabla p = \mathbf{f} \quad \text{Darcy Law}$$
$$\nabla\cdot\mathbf{u} = q \quad \text{Conservation}$$

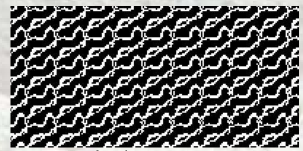
Darcy-Stokes Interface: $\Gamma = \partial\Omega_s \cap \partial\Omega_d$

$$\mathbf{u}_s\cdot\nu_s = \mathbf{u}_d\cdot\nu_s \quad \text{Flux continuity}$$
$$2\nu_s\cdot D\mathbf{u}_s\cdot\tau = -\frac{\alpha}{\sqrt{K}}\mathbf{u}_s\cdot\tau \quad \text{Beavers-Joseph}$$
$$2\mu\nu_s\cdot D\mathbf{u}_s\cdot\nu_s = p_s - p_d \quad \text{Continuity of normal stress}$$

Homogenization and the Macro-Scale Model

Suppose the domain Ω is periodic of period ϵY. Scale K and μ by ϵ^2 and let $\epsilon \to 0$.

Y



$\to|\,\epsilon\,|\leftarrow$

Theorem: The flow converges weakly to the solution of the homogenized Darcy problem

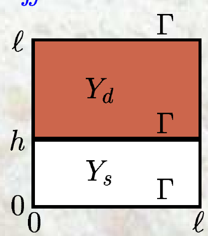
$$\mu\tilde{K}^{-1}\mathbf{u} + \nabla p = \mathbf{f}$$
$$\nabla\cdot\mathbf{u} = q$$

where $\tilde{K}$ is symmetric and positive definite, and

$$\tilde{K}_{i,j} = \frac{1}{|Y|}\left\{\int_{Y_s}\omega^s_{j,i}(y)\,dy + \int_{Y_d}\omega^d_{j,i}(y)\,dy\right\}$$

where $\omega_{j,i}$ satisfies a Darcy-Stokes system on Y.

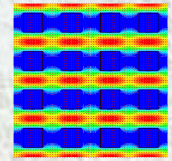
Effective Permeability of a Layered System



$$\tilde{K}_{11} = \frac{h^3}{12\ell} + \frac{\sqrt{K}}{2\alpha\ell}h^2 + \frac{\ell-h}{\ell}K$$
$$\tilde{K}_{22} = \frac{\ell}{\ell-h}K$$
$$\tilde{K}_{21} = \tilde{K}_{12} = 0$$

Along vug channels, $\tilde{K}_{11}$ is mostly Poiseuille flow in the channel ($h^3/12\ell$). Across vug channels, $\tilde{K}_{22}$ is the harmonic average of K and ∞.

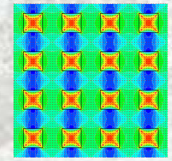
Effective Permeability: Interconnected Vugs



K = 1e-8 cm^2 ($\approx$ 1 darcy)

$$\tilde{K} = \begin{pmatrix} 0.013 & 0 \\ 0 & 0.013 \end{pmatrix}\text{cm}^2$$

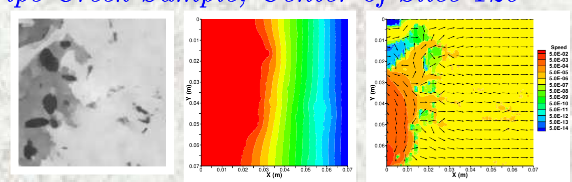
Effective Permeability: Isolated Vugs



K = 1e-8 cm^2 ($\approx$ 1 darcy)

$$\tilde{K} = \begin{pmatrix} \text{1.7e-8} & 0 \\ 0 & \text{1.7e-8} \end{pmatrix}\text{cm}^2$$

Pipe Creek Sample, Center of Slice 120



CT scan | pressure | speed & velocity

Major Conclusions

- Homogenization of the Darcy-Stokes system converges to a Darcy system with an effective permeability $\tilde{K}$ that can be computed.
- Accurate finite element and discontinuous Galerkin methods were developed for the Darcy-Stokes system.
- Preliminary numerical results look promising.

Future Work

- Conduct flow and transport field experiments.
- Model the transport of tracers.
- Verify the accuracy of the macro-scale model for full field-scale simulation.