

Homework problems on the shallow water equations.

1. Derive equation (10) starting from (1), justifying each step.
2. Derive (11) and determine the form of the “differential advection terms.”
3. Starting with the right hand side of (2), derive (12).
4. Consider the 1-D shallow water model:

$$\begin{aligned}\zeta_t + q_x &= 0 \\ q_t + (q^2/H)_x + gH\zeta_x &= 0\end{aligned}$$

where $q = uH$. Assume a flat bottom so $b_x = 0$. Show that we can rewrite this system in quasi-linear form

$$\begin{bmatrix} \zeta_t \\ q_t \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ gH - u^2 & 2u \end{bmatrix} \begin{bmatrix} H_x \\ q_x \end{bmatrix}$$

Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 0 & 1 \\ gH - u^2 & 2u \end{bmatrix}$$