ASE 380P 2-ANALYTICAL METHODS II
EM386L MATHEMATICAL METHODS IN APPLIED MECHANICS II
CSE 386L MATHEMATICAL METHODS IN APPLIED ENGINEERING AND SCIENCES

Exam 2. Monday, Apr 2, 2012

1. (a) Define a complex differentiable function and state the Cauchy-Riemann conditions (5 points)
   (b) Check if the following function is complex-differentiable (15 points)
   $$f(z) = |z|^2 \sin z$$
2. (a) Define branch cuts and select a specific single-value function for
   $$f(z) = \sqrt{1 + \sqrt{z}}$$
   (10 points).
3. (a) State the Laurent Expansion Theorem (5 points).
   (b) Expand the following function into its Laurent series in $2 < |z| < 3$,
   $$\frac{1}{z^2 - 5z + 6}$$
   (15 points).
4. (a) State the Residue Theorem (5 points).
   (b) Use the Residue Theorem to compute the integral:
   $$\int_{-\infty}^{\infty} \frac{2x\, dx}{8x^3 + 1}$$
   (15 points).
5. Consider the initial value problem:
   $$\ddot{x} - x = \delta(t-1), \quad x(0) = 0, \dot{x}(0) = 0$$
   where $\delta(t-1)$ denotes the Dirac's delta acting at $t = 1$.
   (a) Solve first the problem using elementary means (5 points).
   (b) Laplace transform the problem and find the solution in the Laplace domain (10 points).
   (c) Use the Residue Theorem to compute the inverse Laplace transform. Compare the results (15 points).