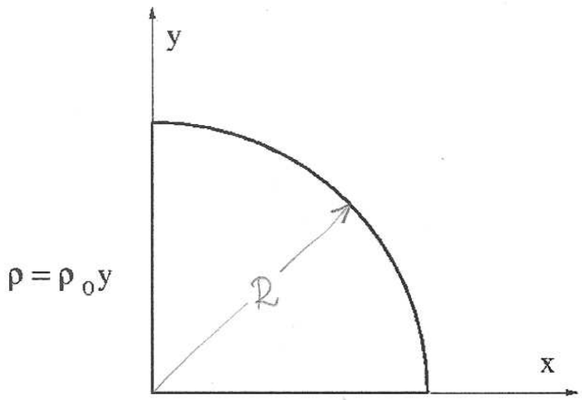
EM311M - Dynamics
Exam 3
Wednesday, May 4, 2011, 6:00-9:00 p.m., ECJ 1.202
Version B

1. Use polar coordinates to compute moment of inertia I_x for a *non-homogeneous* quadrant of a circle shown below. The density $\rho(x, y) = \rho_0 y$ where ρ_0 is a constant. (5 points)



$$I_x = \int_0^R \int_0^{\frac{\pi}{2}} \rho_0 \underbrace{r\sin\theta}_{y} \underbrace{(r\sin\theta)^2}_{y^2} r\,dr\,d\theta$$

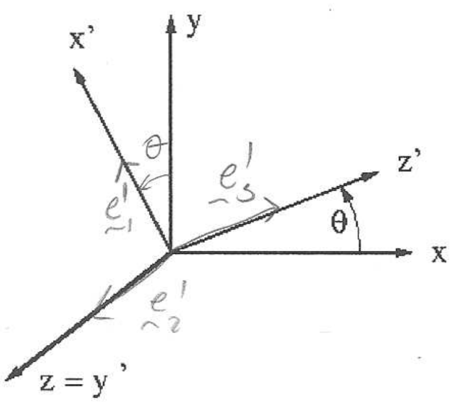
$$= \rho_0 \int_0^R r^4 dr \int_0^{\frac{\pi}{2}} \sin^3\theta \, d\theta$$

$$= \rho_0 \frac{R^5}{5} \cdot \left(-\cos\theta + \frac{1}{3}\cos^3\theta\right)\Big|_0^{\frac{\pi}{2}}$$

$$= \boxed{\frac{2}{15}\rho_0 R^5}$$

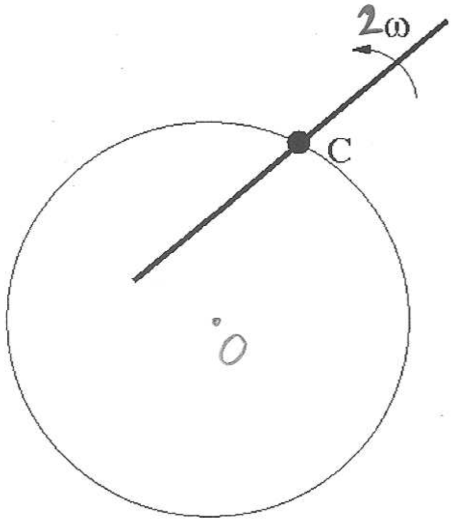
$$\int \sin^3\theta \, d\theta = \int \sin\theta(1-\cos^2\theta)\, d\theta = -\cos\theta + \frac{1}{3}\cos^3\theta$$

2. Define the transformation matrix from system xyz to system $x'y'z'$ and compute it for the systems shown below. (5 points)



$$\underset{\sim}{a} = \begin{pmatrix} -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & \sin\theta & 0 \end{pmatrix}$$

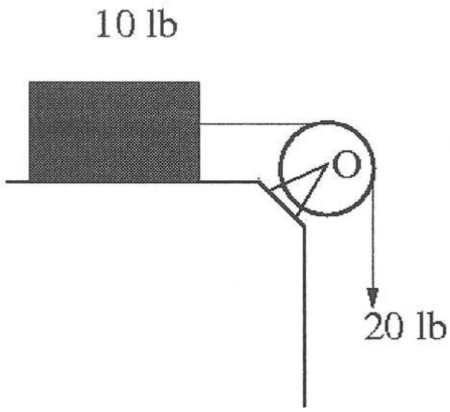
3. Center of mass of a slender bar with mass m and length l is rotating in circle of radius $l/2$ with an angular velocity ω. At the same time, the bar is rotating with respect to its center of mass with angular velocity 2ω. Compute the angular momentum of the bar with respect to the center of the circle. (5 points)



$$H_O = m\omega \frac{l}{2}\frac{l}{2} + \frac{1}{12} m l^2 \, 2\omega$$

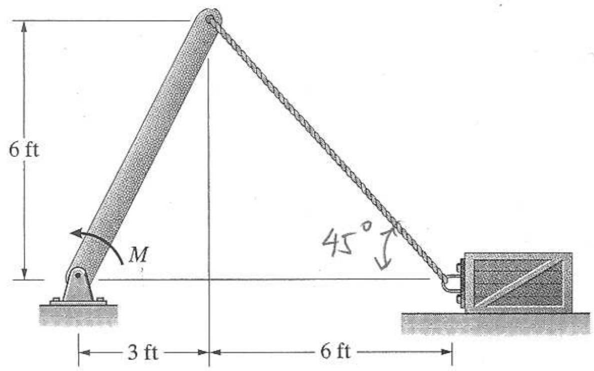
$$= \left(\frac{1}{6} + \frac{1}{4}\right) m l^2 \omega = \boxed{\frac{5}{12} m l^2 \omega}$$

4. The system depicted below starts from rest and the surface is smooth. Consider two cases: case 1: inertia of the pulley is neglible, $I_O \approx 0$, and case 2: inertia of the pulley is significant and must be taken into account, $I_O > 0$. In which case will the acceleration of the block be smaller ? Explain. (5 points)



5. Derive the equation of angular motion for a rigid body undergoing an arbitrary motion, in the body-fitted system of coordinates. You may start from the principle of angular impulse and momentum. (5 points)

6. The slender bar weighs 40 lb, and the cart weighs 80 lb. At the instant shown, the velocity of the crate is zero and it has an acceleration of 10 ft/s^2 toward the left. The coefficient of kinetic friction between the horizontal surface and the crate is $\mu_k = 0.25$. Determine the couple M and the tension in the cable. (25 points)

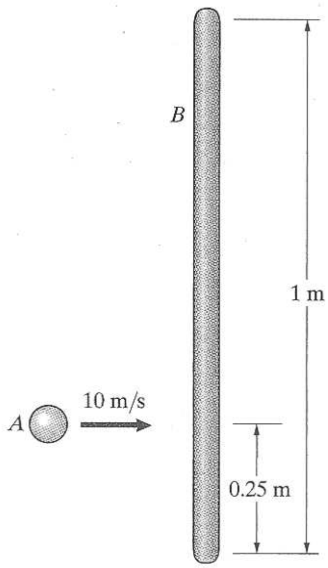


$$M = \frac{1}{3}\frac{40}{32.2}\,45\,\frac{10}{9} + 9\,\frac{0.25\times 80 - \frac{80}{32.2}(-10)}{1+0.25} + \frac{3\times 40}{2}$$

$$= 404\ [\text{ft}\cdot\text{lb}]$$

$$T = \frac{0.25\cdot 80 - \frac{80}{32.2}(-10)}{\frac{\sqrt{2}}{2}(1+0.25)} = 50.7\ [\text{lb}]$$

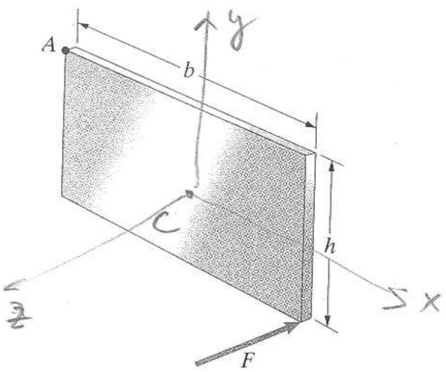
7. The 2-kg sphere A is moving toward the right at 10 m/s when it strikes the unconstrained 10-kg slender bar B. The coefficient of restitution is $e = 0.6$. What is the angular velocity of the bar after the impact? (25 points)



$$v_{Cx}^{a} = \frac{2 \cdot 10 \cdot 1.6}{1.75 \cdot 2 + 10} = \frac{32}{13.5} = 2.37 \left[\frac{m}{s}\right]$$

$$\omega_B^{a} = 3 v_{Cx}^{a} = 7.11 \left[\frac{rad}{s}\right]$$

8. The dimensions of the 20-kg thin plate are $h = 0.6$m and $b = 0.4$m. The plate is stationary relative to an inertial reference frame when the force $F = 40$N is applied in the direction perpendicular to the plate. *No other forces or couples act on the plate.* What is the acceleration of point A at the instant when the force is applied ? (25 points)



$I_x = \frac{1}{12} 20 \; 0.6^2 = 0.6 \; [kg \, m^2]$

$I_y = \frac{1}{12} 20 \; 0.4^2 = 0.267 \; [kg \, m^2]$

$I_z = 0.867 \; [kg \, m^2]$

$\underset{\sim}{a}_c = (0, 0, -2) \; [\frac{m}{s^2}]$

$$\underset{\sim}{M}_c = \begin{array}{r} \underset{\sim}{CB}\,(0.2, -0.3, 0) \\ \times \; \underset{\sim}{F}\,(0, 0, -40) \\ \hline (12, 8, 0) \end{array}$$

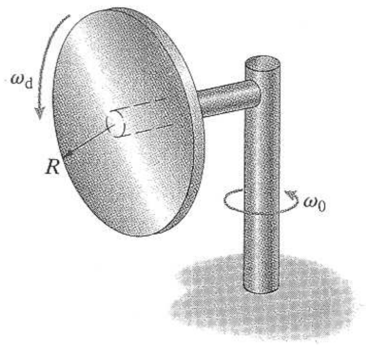
$\underset{\sim}{\alpha} = (20, 30, 0) \; [\frac{rad}{s^2}]$

$$\underset{\sim}{a}_A = (0, 0, -2) + \begin{array}{r} (-20, 30, 0) \\ \times \; (-0.2, 0.3, 0) \\ \hline (0, 0, 12) \end{array}$$

$= (0, 0, 10) \; [\frac{m}{s^2}]$

9 (bonus). The radius of the 10-kg disk is $R = 0.4$m. The disk is pinned to the horizontal shaft and rotates with a constant angular velocity $\omega_d = 4$ rad/s. Determine the magnitude of the couple exerted on the disk by the horizontal shaft.

(25 points)



$$|\underset{\sim}{M}_c| = \frac{1}{2} \cdot 10\ 0.4^2\ \omega_0 \cdot 4 = 3.2\ \omega_0\ [kg \cdot m^2 \cdot s^{-2}]$$